A DYNAMIC MODEL OF CONSTRUCTION OF RETAIL CENTRES IN METROPOLITAN AREAS

Stan Czamanski[1]
111 Hanassi Avenue, #6-A, Haifa 34641, Israel

ABSTRACT A casual observation indicates that construction of new retail centres does not proceed continuously over time. In many metropolitan areas, periods of intensive building activity alternate with spells without any significant additions to existing facilities. These cyclical movements are local and embedded in, but separate from, fluctuations due to business cycles. The model presented tries to cast this phenomenon into rigorous terms offering a working hypothesis and a method of confronting it with data. The analytic framework is inspired by the well known Lotka-Volterra model. The analysis indicates that the existence as well as the length and amplitude of cycles is a function of several parameters some of which are national and some are essentially local.

1. INTRODUCTION

A casual observation indicates that construction of new shopping centres and more generally expansion of retail facilities do not proceed continuously over time. In many metropolitan areas, periods of intensive building activity alternate with spells without substantial additions to existing structures. This phenomenon is even more pronounced in terms of size of facilities per capita or, in other words, of their density. These cyclical movements which are basically local are embedded in, but separate from, fluctuations in the national economy due to business cycles. For example, during the long expansion of the 1980's, there were clearly discernible two periods of increased construction of shopping centres in the Miami Metropolitan area. This phenomenon was even more pronounced at the national level leading to an oversupply of retail space, lowering of rents and raising vacancies (Benjamin and Jud, 1994).

The model presented here tries to cast this phenomenon into rigorous terms offering a working hypothesis and a method of confronting it with data.

2. SOME DEFINITIONS

Disposable income of inhabitants of a metropolitan area is usually defined as:

$$DI = PI - T$$

[1] The author is Professor Emeritus of Regional Science and City Planning, Cornell University, Ithaca NY and Visiting Professor of Economics, Florida International University, Miami FL. He wishes to thank Professor P. Liossatos and an anonymous referee for helpful comments.

where DI = personal disposable income, PI = personal income, and T = personal taxes. Personal disposable income can either be consumed or saved:

$$DI = C + S$$

where C = consumption of goods and services by households, and S = savings.

Another way of looking at personal disposable income is to subtract from it that part which households spend on consumption of basic necessities, such as food (or more generally convenience goods), rent or mortgage payments, school tuition, maintenance of a car or medical expenses. The remaining part of personal disposable income may be called discretionary income:

$$DISC = DI - BAS$$

where $DISC$ = discretionary income, and BAS = expenses on basic necessities. The level of basic expenses of a household while rising slowly over time is fairly constant and is not influenced strongly by fluctuations in disposable income.

After taking care of necessities, households have to decide how to allocate the remaining personal disposable income, or discretionary income, between expenses on shopping goods (which include big ticket items) and savings.

$$DISC = SHOP + S$$

where $SHOP$ = purchases of clothing, washing machines, TV sets, etc.

Finally, some variables may be defined in per capita terms:

X = discretionary income per capita, or ($DISC/P$), where P is population of the metropolitan area;

Z = purchases of shopping goods per capita, or ($SHOP/P$), and

Y = total floor area devoted to shopping goods per capita, which serves as an index of availability of retail facilities.

The presentation so far has been limited to introducing a few definitions and identities. Next, some hypotheses upon which the model is based will be discussed, at first without considering the effects of business cycles.

3. THE CONSUMPTION DECISIONS

In the short-run, with which this study is concerned, the relation between total consumption and current income (C/DI) is non-proportional. Consumption of households may originate, in line with the Life Cycle Hypothesis, either in earned income or in accumulated wealth (Ando and Modigliani, 1963). The intercept of the short-run consumption function measures the effect of wealth. It is not constant over time and the shifting short-run consumption function traces out the long-run consumption function which has no intercept. Since the ratio of wealth to disposable income is changing, the average propensity to consume will change. The influence of wealth upon consumption of households may be the main source of variability of total consumption. In the present study all income levels have been combined, but a future analysis may profitably disaggregate population into three income groups: low, middle, and high, and perform a separate analysis for each. With social security payments adjusted for cost of living increases and with evidence of greater concentration of wealth at the top, the middle class is losing ground.

The household's wealth may be either spent directly or become the source of additional income. In the context of the present study a somewhat stronger assumption is adopted, namely that expenses of households on necessities, such as food, mortgage payments, or school tuition, (*BAS*) are mainly derived from, or are at least proportional to, earned income. Since personal disposable income is identically divided between expenses on necessities and discretionary income it follows that the main source of discretionary income is wealth. Furthermore, over short time periods discretionary income is highly variable.

Next, consider the allocation of discretionary income (*DISC*), or of what is left of disposable income (*DISC* = *DI* - *BAS*), between purchases of shopping goods and savings. The volume of purchases of shopping goods per unit of time is, among other things, a function of the size of discretionary income and of the extent and quality of retail facilities. Putting it in more rigorous terms involves one of the assumptions upon which the model is built, namely that the volume of purchases of shopping goods is proportional to the product of per capita discretionary income and the size of retail facilities, both expressed in per capita terms, or

$$Z = \beta \, (X\,Y) \tag{1}$$

This assumption needs to be slightly elaborated. The purchases of big ticket items change the balance within the discretionary income (*DISC*), by increasing expenses on shopping goods (*SHOP*) and decreasing correspondingly savings (*S*).

Equation (1) could be reformulated as a logistic function by writing

$$Z = \beta X Y (1 - \bar{a} Y / \bar{Y})$$

where $0.5 < \bar{a} < 1$ and $\bar{Y}$ = saturation floor area per capita, obtainable, for example, from the richest city. Some preliminary analysis carried out on data from the Miami metropolitan area show, however, little difference between the two formulations.

In the United States personal savings as a percentage of disposable income (*S/DI*) has been steadily decreasing from 7.8 per cent during 1971-1980 to 4.5 per cent in 1990. Several explanations of the phenomenon have been proposed. The first, based on the life cycle hypothesis, claims that increased social security benefits have reduced the need to save for retirement years. The second puts the blame on enhanced use of credit cards, while the third looks for the reason in the growing number of two income families in which the risk of unemployment is lessened.

Another explanation stresses the role of advertising. Informative and especially persuasive publicity affects the choice of items to be purchased but beyond it generates the desire to buy, frequently exceeding reasonable needs. The welfare effects of modern advertising have been extensively discussed in the literature. Some economists went even so far as to suggest that modern giant corporations, by skilful use of Madison Avenue, can manipulate consumers, generate demand for their products and operate virtually outside of the normal market mechanism (Galbraith, 1967; Nelson, 1978; Nelson, 1981; Shapiro, 1980; Solow, 1989). In the present study it is hypothesized that construction of modern retail facilities, with elaborate displays and exhibits, plays a role similar or complementary to advertising by inducing consumers to increase their discretionary spending.

Next, consider the evolution of personal wealth, since according to the assumption already made, wealth is the source of discretionary income (*DISC*). For most households their wealth is due to accumulated past savings rather than to inheritance, or accidental causes. Savings simply represent the not consumed part of discretionary income. Hence, the rate of growth of personal per capita wealth is:

$$\frac{dw}{dt} = X_t - Z_t \tag{2}$$

where w = wealth per capita.

Now, according to the assumption already made, discretionary income is a linear function or simply a proportion of wealth, or

$$X = a_1 w \tag{3}$$

The rate of growth of discretionary income would then be

$$\frac{dX}{dt} = a_1 \frac{dw}{dt} = a_1 (X - Z) \tag{4}$$

and substituting equation (1) into (4)

$$\frac{dX}{dt} = a_1 X - a_1 \beta (XY) \tag{5}$$

The rate of growth of discretionary income (dX/dt) would, therefore, be a function of two variables, namely size of discretionary income (X) and size of shopping facilities (Y). Both variables will usually increase over time, since with the passage of time households normally save more, while shopping expenses might be subject to fluctuations over time, partly reflected in the coefficient (β).

Defining, in order to simplify notation, $b_1 \equiv a_1 \beta$ we arrive at

$$\frac{dX}{dt} = a_1 X - b_1 (XY) \tag{6}$$

These assumptions find some confirmation in macroeconomic data. Over time the national disposable personal income is increasing steadily, both globally and in per capita terms, except for fluctuations due to business cycles. This trend would be discernible also in a metropolitan area, although it might be distorted by strong rural-urban, or interurban migration, which can change the composition of the population and the structure of incomes. Basic expenses (*BAS*) per capita also increase over time but more slowly, hence the rate of increase of discretionary income which is the difference between the two, both globally (*DISC*) and in per capita terms (X), is higher than that of personal disposable income.

4. ADDITIONS TO RETAIL FACILITIES

The construction of shopping centres in a metropolitan area or the rate at which they are expanding is a function of the degree to which the demand for such facilities is satisfied. The latter is a positive function of the demand for shopping goods, already defined as proportional to the product of the size of shopping facilities and of discretionary income, and a negative function of the size of existing facilities for marketing shopping goods. In the absence of demand, the size of such facilities would normally decrease in absolute terms due to normal wear and tear and above all due to obsolescence related to the appearance of new forms of retailing. The decline would be more pronounced in per capita terms in view of the steady growth of population of metropolitan areas. Thus

$$\frac{dY}{dt} = -a_2 Y$$

Taking into consideration the second element, namely increase in demand as defined in equation (1), the rate of growth of marketing facilities for shopping goods can be expressed as:

$$\frac{dY}{dt} = -a_2 Y + b_2 (XY) \tag{7}$$

The analysis of the time path of expansion of marketing facilities for shopping goods has been inspired by the well-known Lotka-Volterra model (Lotka, 1956; Volterra, 1931). First, one could combine equations (6) and (7) by eliminating between them *dt*, but instead we shall proceed differently and multiply equation (6) by (a_2/X) yielding:

$$\frac{a_2}{X}\frac{dX}{dt} = a_2 a_1 - a_2 b_1 Y$$

and equation (7) by (a_1/Y) yielding:

$$\frac{a_1}{Y}\frac{dY}{dt} = -a_1 a_2 + a_1 b_2 X$$

Adding these last two equations results in equation

$$a_2 \frac{d\log X}{dt} + a_1 \frac{d\log Y}{dt} = -a_2 b_1 Y + a_1 b_2 X \tag{8}$$

Next, multiply equation (6) by b_2 and equation (7) by b_1 and add them. This will yield:

$$b_2 \frac{dX}{dt} + b_1 \frac{dY}{dt} = a_1 b_2 X - a_2 b_1 Y \tag{9}$$

The right-hand sides of equations (8) and (9) are equal, hence the left-hand sides have to be equal too. Writing this out and collecting terms yields:

$$- a_2 \frac{d\log X}{dt} - a_1 \frac{d\log Y}{dt} + b_2 \frac{dX}{dt} + b_1 \frac{dY}{dt} = 0 \qquad (10)$$

Integrating this last equation, one obtains:

$$- a_2 \log X - a_1 \log Y + b_2 X + b_1 Y = A \qquad (11)$$

where A is the constant of integration. Setting $k \equiv e^{A}$ and taking exponents yields:

$$e^{b_2 X} e^{b_1 Y} X^{-a_2} Y^{-a_1} = k \qquad (12)$$

or collecting terms:

$$(X^{-a_2} e^{b_2 X}) = k\,(Y^{a_1} e^{-b_1 Y}) \qquad (13)$$

and defining $g(X) = (X^{-a_2} e^{b_2 X})$ and $f(Y) = (Y^{a_1} e^{-b_1 Y})$ one can write:

$$g(X) = k f(Y) \qquad (14)$$

Next, consider the form of the two functions *g(X)* and *f(Y)* by taking their derivatives. The first derivative has the form:

$$\frac{dg(X)}{dX} = -a_2 X^{-a_2 -1} e^{b_2 X} + b_2 X^{-a_2} e^{b_2 X} = g(X)\,[b_2 - \frac{a_2}{X}] \qquad (15)$$

This equation takes the following values:

for $X = \frac{a_2}{b_2} \rightarrow \frac{dg(X)}{dX} = 0$

for $0 \leq X < \frac{a_2}{b_2} \rightarrow \frac{dg(X)}{dX} < 0$

for $X > \frac{a_2}{b_2} \rightarrow \frac{dg(X)}{dX} > 0$

Since the second derivative has no real roots, the graph of this function is as shown in Figure 1. The first derivative of the function *f(Y)* is:

$$\frac{df(Y)}{dY} = a_1 Y^{a_1 -1} e^{-b_1 Y} - b_1 Y^{a_1} e^{-b_1 Y} = f(Y)\,[a_1 Y^{-1} - b_1] \qquad (16)$$

This equation takes the following values:

for $Y = \frac{a_1}{b_1} \rightarrow \frac{df(Y)}{dY} = 0$

$$\text{for } 0 \leq Y < \frac{a_1}{b_1} \rightarrow \frac{df(Y)}{dY} > 0$$

$$\text{for } Y > \frac{a_1}{b_1} \rightarrow \frac{df(Y)}{dY} < 0$$

The second derivative has real roots and there are two points of inflection which for the sake of simplicity may be omitted. Its graph is shown in Figure 2.

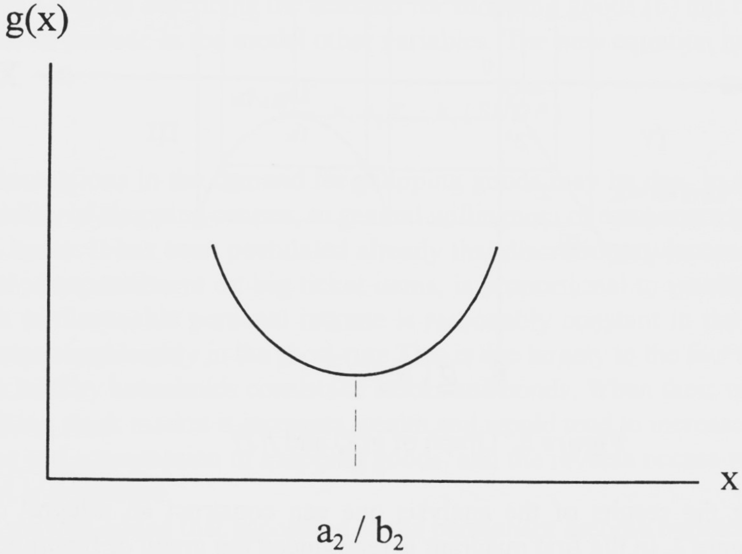

Figure 1. Graph of $g(X) = (X^{-a_2} e^{b_2 X})$

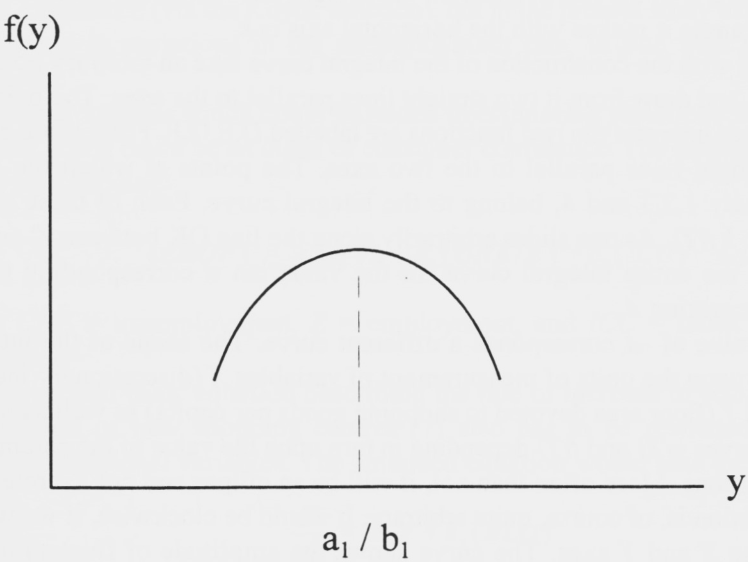

Figure 2. Graph of $f(Y) = (Y^{a_1} e^{-b_1 Y})$

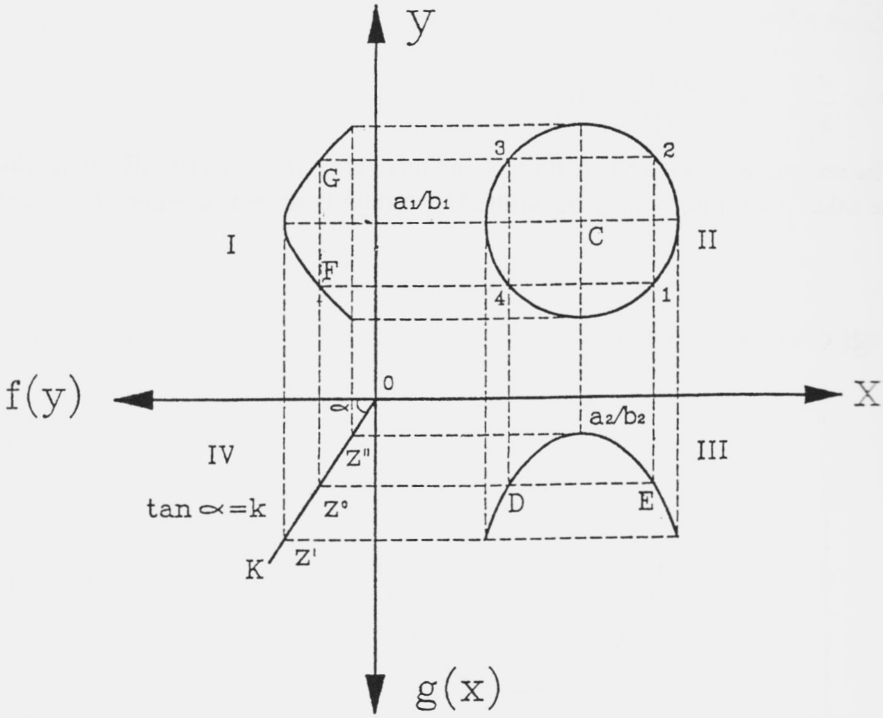

Figure 3. Graph of *g(X)* and *f(Y)*

Combining the results of the analysis one can construct an integral curve presented in Figure 3. In the first quadrant is reproduced the graph of function *f (Y)* from Figure 2, turned 90°. In the third quadrant is reproduced the graph of function *g (X)* from Figure 1. In the fourth quadrant the straight line represents equation (14). The tan of the angle it makes with the horizontal axis is *k*.

To proceed with the construction of the integral curve take an arbitrary point Z° on the line OK and draw from it two straight lines parallel to the axes. The points at which these lines intersect the two functions are labelled D,E,G,F. From those points are drawn straight lines parallel to the two axes. The points at which the latter intersect, namely 1,2,3 and 4, belong to the integral curve. Each of these points equates *g(X)* to *k f(Y)*. As one slides arbitrarily along the line OK between Z' and Z" one can draw the entire integral curve for the value tan α corresponding to the constant of integration *A*.

To each value of *A* corresponds a different curve. The shape of the integral curve depends upon the units of measurement of variables *X* (discretionary income per capita) and *Y* (floor area devoted to shopping goods per capita) as well as on the shape of the curves *g(X)* and *f(Y)* depending in turn upon the value of the parameters of the two functions. More often than not, it will be an ellipse and not a circle. The direction of motion is, of course, quite arbitrary. It would be clockwise, if we would interchange the *X* and *Y* axes. The curve shows the amplitude of fluctuations in expenses on shopping goods, on the one hand, and fluctuations in construction of new retail facilities on the other. The state if equilibrium is the singular point whose coordinates are ($Y = a_1/b_1$) and ($X = a_2/b_2$) (Gandolfo, 1980).

5. EFFECTS OF BUSINESS CYCLES

Fluctuations in demand for shopping goods and in the construction of retail facilities are not independent of the influence of business cycles, which are usually much more pronounced. The next step is thus to incorporate the effects of additional factors into the model so far developed.

The equation describing the demand for shopping goods (6) has to be rewritten in order to include in the model other variables. The new equation has the form:

$$\frac{dX}{dt} = a_1 X - b_1 (SHOP) \tag{17}$$

Fluctuations in the demand for shopping goods may be due, in addition to the availability of shopping centres, to general willingness of consumers to spend on big ticket items. It has been postulated already that discretionary income which is the source of expenditures on big ticket items, is proportional to wealth. The ratio of wealth to disposable personal income is reasonably constant in the long-run, but fluctuates considerably in the short-run. This is due largely to the fact that part of the wealth held by households consists of stocks and bonds. When their value is high in a booming stock market it increases wealth and would tend to increase discretionary income and consumption of shopping goods, and the reverse occurs when the stock market is depressed.

It may be added that according to the permanent income hypothesis, the main factor influencing the volume of consumption are expectations concerning the fixed component of households income (Friedman, 1957). These expectations are based on past experience (via the Koyck function), but in the present study expectations are summarized in variations in the unemployment rate, in line with the life cycle hypothesis.

Another variable which could be added in estimating variations in expenses on shopping goods could be an index of consumer confidence. The estimating equation would then become:

$$(SHOP) = a_3 (XY) - b_3 (UNE/E) + c_3 (ICC) \tag{18}$$

where *UNE* = unemployment, *E* = employment, and *ICC* = index of consumer confidence.

The second basic equation describing the rate of increase of retail facilities or construction of new shopping centres (7) also has to be rewritten, in order to introduce additional variables. The amended equation would take the form:

$$\frac{dY}{dt} = a_2 Y + b_2 (BLD) \tag{19}$$

where *BLD* = volume of construction of shopping centres.

Investments in the construction of new shopping centres may be assumed to be a function of the general business climate, finding its expression in changes in the rate of growth of the gross domestic product, prevailing interest rates and availability of credit and government incentives, such as changes in investment tax rates. The estimating equation would then take the form:

$$(BLD) = a_4(SHOP) + b_4GDP + c_4i + d_4(AF) \quad (20)$$

where *GPD* = gross domestic product, *i* = interest rate on mortgages, and *AF* = index of availability of credit and government incentives.

Together equations (17), (18), (19) and (20) form an identifiable system, since both the rank and order conditions are met.

REFERENCES

Ando, A. and Modigiliani, F. (1963) The life cycle hypothesis of saving: applications and tests. *American Economic Review*, 55, pp. 55-84.

Benjamin, J. D. and Jud, G. D. (1994) An analysis of shopping centre investment. *Journal of Real Estate Finance and Investment*, 10, pp. 161-168.

Friedman, M. (1957) *A Theory of Consumption Function.* Princeton University Press: Princeton, NJ.

Galbraith, K. (1967) *The New Industrial State*. Houghton Mifflin: Boston, MA.

Gandolfo, G. (1980) *Economic Dynamics: Methods and Models*. (2nd ed.) North Holland Publishing Co.: Amsterdam.

Nelson, P. (1978) Information and consumer behavior. *Journal of Political Economy*, 78, pp. 311-329.

Nelson, P. (1981) Advertising as information. *Journal of Political Economy*, 81, pp. 729-754.

Lotka, A. (1956) *Elements of Mathematical Biology*. Dover Publications reprint: New York, NY.

Shapiro, C. (1980) Advertising and welfare: comment. *Bell Journal of Economics*, 11, pp. 749-752.

Solow, R. (1989) The new industrial state or son of affluence. *Public Interest*, 9, pp. 100-108.

Volterra, V. (1931) *Lecons sur la theorie mathematique de la lutte pour la vie*. Gauthier-Villars: Paris.