THE ROLE OF REGIONAL INTERACTION IN REGIONAL GROWTH: Competition and Complementarity in the U.S. Regional System¹

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STRACT The role of regional interaction in multi-regional growth is examined by imaging the parameters for a discrete nonlinear model of relative dynamics with maximum bood methods. This model specification approaches the issues of spatial dependence in interent form from the methods that have been used to date. It provides evidence for the of multi-regional growth as a zero-sum game, in which a mixed pattern of competition complementarity exists among non-contiguous regions. The approach is illustrated with agregate set of regions for the U.S. economy and forecasts are made of the progress of convergence into the next century. It is suggested the methodology might prove to a seful alternative to the usual method of incorporating exogenous changes into regional methods.

I INTRODUCTION

Do regions compete or complement each other? Can these types of regional eractions coexist in a national economy? If so, do they fulfill the "first law of graphy" in relation to the effects of neighbourhood or close spatial association? approaches may be needed to address these questions. This paper examines one model proposed recently by Dendrinos and Sonis (1990) for modelling the timemodel of a single statistical population over multiple locations. It is applied to

The detailed comments and suggestions of Peter Dixon and the referees are very much

an examination of the role of regional interaction in the growth of the U. S. regions. The parameters are obtained by classical maximum likelihood estimation of a loglinear specification; furthermore, the parameter signs and significance can be interpreted as characterizing the regional patterns of competition and complementarity.

The stimulus for this work was provided by increasing dissatisfaction with the way in which traditional econometric, input-output and linked models incorporate exogenous change into a specific regional model. For example, a typical econometric model for a region would use a national econometric 'driver'; however, for many sectors of the Chicago regional economy, the linkages are far stronger with specific parts of the U.S. with the result that a great deal of potential explanation is lost when the exogenous data are specified for the U.S. as a whole rather than for specific subsets. When considering these issues with reference to Chicago (or any other region), a set of questions may be raised:

- How does the region's degree of self-sufficiency compare with other regions in the U.S. (and, for that matter, in the rest of the world)?
- Given the degree of interaction with the rest of the U.S., which parts of the externalto-region world within U.S. borders benefit the most from this interaction?
- What pattern of regional interaction emerges when the system of North American regional economies are considered? What implications does this pose for Chicago, in the context of the North American Free Trade Area?
- How can this information be used to better understand the nature of interregional trade in the U.S. and to provide more accurate forecasting systems for regional economies?

The transformations that continue to evolve in the U.S. economy have modified significantly the nature and distribution of economic activity. Disparities in per-capita incomes (Amos, 1988), differences in the patterns of regional specialization, and organizational and technological changes at the firm level are all facts that call for a new approach to examine the way in which national economies function. As recent research suggests, the dominant trends of national growth are often not at all representative of the fluctuations taking place at the regional (sub-national) level in the U.S. These questions provide the major motivation for the present paper; the work of Henderson and Krueger (1965) provided some initial stimulation in that their forecasting and impact model for the Upper Midwest divided the external-to-the-region world into a nine-fold partition, providing the capability for each external region to exert a different influence upon the Upper Midwest.

The major objective is to provide an empirically based and tested framework for understanding spatial interactions among U.S. regions, so that the structural changes taking place in these interactions can be examined. In addition, this framework can be used in enhancing the predictive ability of regional econometric/input-output models of the kind represented by the Chicago Region Econometric Input-Output



Figure 1b. Proposed System

Model (**CREIM**)². At the present time, most regional models exemplify the characterization of Bolton (1985) as 'top-down' (see Figure 1a). Essentially, they are 'two-region' models: the region of interest and the nation. The goal of this research is to build an alternative framework; here, the nation is replaced by a space-time system for a finite division of the U. S. economy into individual states or census regions (see Figure 1b).

With such a specification, it is hoped that the richness of our understanding of spatial interaction and interdependencies can be harnessed for the provision of more sensitive modelling and forecasting of individual regions. The approach chosen is still parsimonious in model scope and certainly avoids any tendency towards *data avaricious* models whose grand design hides the impossibility of their empirical implementation.

As a by-product of this research, it is hoped that a better understanding of the nature of the space-time regional economic system in the U. S. will be revealed. Opportunities to provide the basis for a set of taxonomies of regional economy-types, for growth and development trajectories or a series of *stylized facts* in the tradition of Johansen (1960) and Chenery and Syrquin (1975) could be explored.

2. ANTECEDENTS

Econometric modelling of space-time processes requires dealing with the problem of spatial effects. These effects are known as spatial heterogeneity (heteroskedasticity) and spatial dependence (autocorrelation). The various methods devised for dealing with these problems involve the use of a matrix of weights describing regional contiguity patterns or network linkages. This scheme underlies the three models of spatial econometrics: weighted OLS, GLS and Spatial Seemingly-Unrelated Regression (SSUR).

CREIM was developed in cooperation with Richard Conway whose Washington State projection and Simulation Model (Conway, 1979, 1990, 1991) has been in use for many years.

Use of the weight matrix stems from the acceptance of the first law of geography, whereby there would be an expectation of observing a greater correlation between series describing processes in regions that are closer in space. But nothing guarantees that the interactions picked by the weight matrix represent the complete set of significant links between the regions studied that are influenced by spatial effects. This is one of the major research challenges in the spatial econometrics literature (Anselin, 1988, p. 254). A model specification that is free from the explicit incorporation of these effects would contribute to the liberation of the econometric modelling of space-time processes from using these techniques, while addressing adequately the research questions formulated in Section 1³.

The traditional views on regional growth (Richardson, 1978, pp. 145-146) state that growth in regional output is either (a) a zero-sum game, and thus growth in a region can take place only in expense of growth in another region, or (b) generative, implying that an efficient organization of production within each region will lead to increased regional and national growth in output. In the first case, regional interaction plays a fundamental role in determining regional growth and it is possible to find a mixed pattern of regional competition and complementarity. In the second case, regional growth seems to be determined also by processes endogenous to the region.

These views may be reconciled if empirical work is able to show that it is *how* the dynamics of sectoral interactions *within* the regions react to the regional interaction with other regions that determine the patterns of competition and complementarity in the multi-regional system. This paper examines only the last part of this hypothesis, to explore if indeed there exists a pattern of regional interaction with a specific econometric structure. Further work will be needed in (a) a multiple time series framework and (b) a multisectoral specification to examine the role of the sectoral dynamics internal to regions in determining the observed patterns of regional interaction.

3. DISCRETE RELATIVE NONLINEAR DYNAMICS

Let Γ_{sT} be an economy defined over space and time. The indices *S* and *T* indicate the finite number of *regions* in the economy and a finite time horizon respectively. Regional economic activity within the nation is represented by an *S*-dimensional vector $X_t' = (X_{1t}, X_{2t}, \dots, X_S), [0 < X_{st} < 1; s = 1, \dots, S; t = 1, \dots, T]$. Given the data available, this vector is defined as a vector of Gross Regional Products (GRPs), at constant prices of 1982.

The main issue centres on the most efficient way to examine the pattern of regional interaction in the context of the global (national) dynamics of Γ_{sT} .. To answer this question, the behaviour of the GRP stocks needs to be studied by expressing their values *relative* to GNP:

³ However, as one of the reviewers has correctly pointed out, there is no guarantee that spatial dependence will be eliminated by expressing variables with respect to a numeraire; the empirical problem is the degree to which this process affects dependence. Is it merely a re-scaling process or one that incorporates other properties?

$$Y_{st} = \frac{1}{\sum_{s} X_{st}} X_{st} \qquad [0 < Y_{st} < 1; s = 1, ..., S; t = 1, ..., T]$$
(1)

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where X_{st} , Y_{st} are the absolute and relative values of gross national product in region s at time period t.

The D-S model (after Dendrinos and Sonis, 1990) represents these relative dynamics a discrete nonlinear process of the form:

$$Y_{1t+1} = \frac{1}{1 + \sum_{j} F_{jt}} \qquad [j = 2, 3, ..., S]$$

$$Y_{st+1} = \frac{F_{st}}{1 + \sum_{j} F_{jt}} \qquad [j = 2, 3, ..., S]$$

$$1 \qquad [s = 1, 2, ..., S]$$
(2)

$$\frac{Y_{st+1}}{Y_{1t+1}} = F_{st} \qquad [s = 2, 3, ..., S]$$

Here $\sum_{st} Y_{st} =$

the first region is considered as a numeraire (reference) region. The regions are marked by their shares of national GNP in order to make the choice of the numeraire region; it was decided to use the region with the smallest share of GNP, the Rocky Mountain region, as the numeraire. In this study the log-linear specification of the function F_{st} will be used. Dendrinos and Sonis (1990) define this function as:

$$F_{st} = A_s \prod_k Y_{kt}^{a_{sk}} \qquad [F_{st} > 0; s = 2, ..., S; k = 1, ..., S]$$
(3)

where $A_s > 0$ represents the locational advantages of all regions $s \in S$ and:

$$a_{sk} = \frac{\partial lnF_{st}}{\partial lnY_{kt}} \qquad [s = 2, 3, ..., S; k = 1, 2, ..., S]$$

$$\tag{4}$$

are the interregional growth elasticities, with $-\infty < a_{sk} < \infty$.

In log-linear form, (2) and (3) can be written as:

$$\ln Y_{st+1} - \ln Y_{1t+1} = \ln A_s + \sum_{k=1}^{s} a_{sk} \ln Y_{kt} \qquad [s = 2, ..., S; t = 1, ..., T]$$
(5)

At this level of aggregation, regional interaction is dominated by the pursuit of each region's increase in its share of GNP, which is accomplished by improving their comparative advantages. This improvement depends upon the behaviour of the rest

of the regions, which is taken into account by the sign and value of each a_{sk} (composite growth elasticities), given the locational comparative advantage factor A_s . Negative composite elasticities would identify competing regions and positive elasticities would identify complementary regions. Existence of both types of interaction would be evidence for the interpretation of regional growth as a zero-sum game, in which growth in any y_s would be in expense of the growth of at least one other y_i ($j \neq s$) in Γ_{sr} .

Predictions of the *level* of the regions' GRPs are obtained by multiplying Y_{t+1} by corresponding forecasts of the Gross National Product (obtained from exogenous sources). These forecasts are expected to reveal the (bounded) cyclical deviations of the GRPs from the dominant trend of GNP growth, for the discrete map of relative nonlinear dynamics has been shown to display several different types of dynamics, including stability, periodicity and chaos (cf. Dendrinos and Sonis, 1990: Part III). Given the limited time series available, it would not be possible to extend this analytical framework to the complete set of U.S. states; in this case, some of the ideas and methods that exploit notions of hierarchy and feedback loops would need to be considered (see Sonis, Gazel and Hewings, 1995). A similar problem was faced by White and Hewings (1982) in applying spatial seemingly unrelated regression analysis to modelling employment within a multiregional context within a state.

4. MAXIMUM LIKELIHOOD ESTIMATION

The transformations performed to the series of GRPs as required by the specification of the D-S model determine that its parameterisation is free from spatial effects or other sources of structural instability. In particular, (a) the model is free from the influence of the global process of GNP growth, as the series have been deflated by GNP when expressed in relative terms, and (b) the model does not specify an *a priori* expectation about spatial heterogeneity and dependence. In essence, the model explores the degree to which the regions interact through their competition for *relative* shares of GNP. It is not clear whether such a process avoids the problem of spatial dependence (numeraire) in equations 3 and 5. If this is the case, the pattern of regional interaction can emerge from estimation of the D-S model without using a weight matrix.

Rewriting the rows of equation (5) in matrix form:

where:

$$y_j = X\beta_j + e_j \qquad j = 2, \dots, S \tag{6}$$

$$y_{j} = \begin{bmatrix} \ln Y_{j2} - \ln Y_{12} \\ \vdots \\ \ln Y_{jt} - \ln Y_{1t} \\ \vdots \\ \ln Y_{jT} - \ln Y_{1T} \end{bmatrix}_{(T-1) \times 1}^{(T-1) \times 1}$$

$$X = \begin{bmatrix} 1 & \ln Y_{11} & \ln Y_{21} & \cdots & \ln Y_{s1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \ln Y_{1t} & \ln Y_{2t} & \cdots & \ln Y_{St} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \ln Y_{1T-1} & \ln Y_{2T-1} & \cdots & \ln Y_{ST-1} \end{bmatrix}_{(T-1)\times(S+1)}$$
$$\beta_{j} = \begin{bmatrix} \ln A_{j} \\ a_{j1} \\ \vdots \\ a_{jS} \end{bmatrix}_{(S+1)\times 1}$$

provides the elements for its maximum likelihood (ML) estimation. Standard references (such as Cramér, 1986; Judge *et al.*, 1988) provide the formulation of the ML estimator for the normal case. Estimation of the log-linear D-S model requires that a log-normal distribution be used; details are provided in an appendix.

In specifying the model, important issues related to misspecification (due to simultaneity) were not addressed explicitly although several alternative forms of the equations were tried. By adopting a relative dynamics view of the processes of change, it is hoped that some of these problems can be minimised. The results suggest that these problems are neither trivial nor easy to accommodate.

5. EMPIRICAL RESULTS

Charts 1 through 8 provide some background perspective about the behaviour of the macro (census) regions over the period 1964 through 1988. Two variables are plotted on each chart, the rate of growth of gross regional product and the growth of the regional share of gross national product. These charts should be read in comparison the summary information provided in Chart 9. Here, the regional shares of gross tational product have been displayed for the same time period. During this period, the Great Lakes region experienced a decrease in its share of national product even though, during much of the period, the rate of growth of regional product was positive. Obviously, this region's performance was outstripped by other regions such as the Southeast. Note also the varying degrees to which the recessions of the early 1970s and 1980s affected regional performance. The Southeast and Southwest experience only one negative growth rate in GRP in 1980s while the Great Lakes region had three the sum on the regions and it is in Table 1 that these relationships are explored.

Table 1 summarizes the parameter values obtained from fitting the D-S model the logarithm of the series of U.S. GRPs relative to the U.S. GNP. A pattern of regional interaction has emerged that links non-contiguous regions. This confirms the concern expressed in Section 1 on the generality of the first law of geography.

















	T	able 1. Re	gression Re	sults of Lo	ng-Linear S	pace-Time	Model of R	elative Dyn	namics		
	New	Mid	Great	Great	South	South	Rocky	Far		\mathbb{R}^2	Durbin-
	England	East	Lakes	Plains	East	West	Mountain	West	Constant	Adj	Watson
New England (t-ratios)	0.5202 (-0.4319)	-2.1077 (-0.9906)	-2.5073 (-1.1292)	0.1139 (-0.1030)	-2.2878 (-1.1810)	-1.0491 (-0.8275)	-1.3317 (-2.2148)	-1.3812 (-0.8761)	-18.5500 (-0.8037)	0.9616	1.5846
Mid East (t-ratios)	0.4553 (-0.5815)	-0.1906 (-0.1491)	-1.2312 (-0.9102)	0.6915 (-0.8742)	-1.3464 (-1.1738)	-0.2565 (-0.3183)	-1.2054 (-3.2986)	-0.7167 (-0.7593)	0.0033 (-0.3991)	0.9889	1.8890
Great Lakes (t-ratios)	0.8561 (-0.6557)	3.3855 (-1.5590)	2.4885 (-1.0779)	1.1644 (-1.0784)	1.6012 (-0.8484)	1.0678 (-0.7715)	-0.2229 (-0.3666	2.4901 (-1.5362)	25.9310 (-1.0907)	0.9793	2.1895
Plains (t-ratios)	0.1754 (-0.2484)	0.1774 (-0.1539)	-0.0071 (-0.0059)	0.2049 (-0.2872)	-0.4749 (-0.4591)	-0.0486 (-0.0669)	-0.5518 (-1.6744)	-0.0049 (-0.0057)	0.5555 (-0.0456)	0.9775	1.8684
South East (t-ratios)	-0.1435 (-0.2047)	0.3142 (-0.2700)	-0.6930 (-0.5610)	0.0553 (-0.0988)	0.3166 (-0.3178)	-0.2077 (-0.2784)	-0.8081 (-2.4926)	0.3099 (-0.3559)	-1.2271 (-0.0966)	0.9469	2.0281
South West (t-ratios)	-0.7915 (-1.2197)	-1.0595 (-0.9999)	-1.4161 (-1.2630)	-0.2589 (-0.3948)	-1.1060 (-1.1632)	0.0206 (-0.0309)	-1.1448 (-3.7794)	-0.5933 (-0.7584)	0.0000 (-1.0713)	0.6736	1.9996
Far West (t-ratios)	1.4530 (-1.8260)	1.1348 (-0.8565)	1.3743 (-0.9768)	1.0800 (-1.5815)	1.3111 (-1.1280)	0.4859 (-0.5786)	-0.2264 (-0.6068)	1.7041 (-1.7296)	18.4780 (-1.2708)	0.9540	1.9152
Note: Numeraire	of dependent	t variable: Ro	ocky Mounts	uin.							

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This indicates that given (a) the absence of spatial heterogeneity or dependence, (b) developed transport and communications networks, and (c) the differences in the patterns of regional specialization, use of regional contiguity patterns would provide an incomplete picture of the extent of the interdependence with regions other than the nearest neighbours. While not all the variables are statistically significant, the relationships provide some indications of the direction of the interactions. Table 2 provides a qualitative interpretation of the results; the formal relationships have been reduced to signs and, in the second part of the table, a suggested hierarchy of relationships is proposed on a complementarity-competition scale. The Great Lakes (GL) and Far West (FW) regions exert strong complementarity relationships with the rest of the states but the relationship of the other states with these regions is predominantly negative. For the other regions, the relationships are more varied. However, the relationships would be consistent with a priori expectations; the regions that are dominant producers of intermediate goods and services seem to be positively linked with other regions. The next stage in the investigation would be to explore the components of gross national product and sectoral disaggregation; it is at this, more detailed level, that greater insights into the nature of the relationships will be revealed.

The previous points are more pertinent when the highly asymmetric nature of the econometric structure uncovered is noticed. This is evidenced by some regions' competition as suppliers to other regions while complemented by the same regions when demanding from them. This is sufficient evidence for the existence of a simultaneous pattern of competition and complementarity among the U.S. regions. It implies that policies affecting positively the growth of specific regions' share of GNP will increase or decrease the corresponding dependent region's share of GNP according to the competitive or complementary relationship between them, as indicated by the sign of the corresponding growth elasticities. However, many of the individual coefficient estimates would not survive rigorous *t*-test conditions for their significance; as a result, the analysis must be presented as a first experiment rather than the basis for conjecture about the *exact* nature of interdependencies across space. However, the results presented here complement those of Sonis, Gazel and Hewings (1995) in the context of an examination of the feedback effects generated by interregional trade. These results become even more important when consideration of the nature of externalto-the-U.S. linkages of these regions are taken into account; for example, the Great Lakes, Far West and Mid East regions export between 40-50% of their international exports to Canada, Asia and Europe respectively. These external linkages portend a degree of interdependence with the world economy that would produce a complex web of intra-U.S. interdependencies of the kind suggested by Table 1.

Charts 10 and 11 indicate that, in effect, the model is able to predict the bounded fluctuations of the cyclical changes in the growth of the regional shares, just as was observed during the period (1963-89) and displayed in Chart 12. Note that the vertical axis of Chart 10 shows the *rates of growth* of the shares, not the absolute changes in the share values. Chart 11 highlights three regions that appear to be very interdependent; even here, the rates of growth of the predicted shares are out of phase through 2020. It seems that in the long run, the regional shares will converge to a steady state level fairly close to the predictions of the model. In addition, the accuracy of the predicted

	NE	ME	GL	GP	SE	SW	RM	FW	+	-
NE	+	-	-	+	-	-	-	-	2	6
ME	+	-	-	+	-	-	-	-	2	6
GL	+	+	+	+	+	+	-	+	7	- 1
GP	+	+	-	+	-	-	-	-	3	5
SE	-	+	-	+	+	-	-	+	4	5
SW	-	-	1	-	-	+	-	-	1	7
FW	+	+	+	+	+	+	-	+	7	1
+	5	4	2	6	3	3	0	3		
-	1	3	5	1	4	4	7	4		
			(b)	Qualita	tive or	lering				

Table 2.	Qualitative Analysis of the Competitive/Complementary Relationships
	(a) Qualitative Relationships



Complementarity
Competition

regional shares can be assessed from Chart 12, which presents the percent deviation of the predicted from the observed regional shares. There is some suggestion that the model's deviations from observed shares has increased over the observed time period even though the forecasts (Chart 10) indicate convergence. Until a longer time series of data is available, it will be difficult to know whether the dampening process implied in Chart 10 is indicative of a real convergence towards stable shares. The experience of the 1990s recession suggests that U.S. macro regions are still not marching in lock-step with the nation; however, the major concern is whether this is a short-term deviation from a longer-term trend towards convergence of behaviour. Note, also, that the cyclical behaviour (Chart 11) for three regions still suggests considerable differences in the growth rates for the shares in terms of timing and amplitude for the period 1987 through 2020 before a significant dampening-out process begins. In the historical period (Chart 12), the year-to-year fluctuations are far more varied than during the forecast period since the D-S model tends to smooth out fluctuations

6 CONCLUSIONS AND DIRECTIONS FOR FURTHER WORK

This paper has uncovered an asymmetric pattern of competition and complementarity mong the U.S. regions, using a nonlinear dynamic functional form that is endogenised







the spatial interaction effects and that reveals bounded cycles of fluctuating growth over time. The results presented in Charts 10 and 11 point clearly towards the significance of the lag structure over space and time among the U.S. regions in a multiple autoregressive moving-average (ARMA) framework. Another important step in this research is to provide an empirical explanation for the patterns of competition and complementarity uncovered here. This can be accomplished by moving from the macro level of this paper to the *meso* level, i.e. by estimating the D-S model with the series of regional sectoral outputs, in order to determine what elements in the interactions among the sectoral production of regions determine the relationships at the macro level found in this paper.

Moreover, further work follows from the contradictions between the assumptions underlying the ML estimator derived and those of the D-S model, which carries two fundamental assumptions: relativity and interdependence. Relativity means that the model is predicting the dynamics of GRPs relative to the global dynamics of the national economy, based on a sample of non-experimental data. Classical econometrics' asymptotic results for parameter estimation assuming a repeated-sampling data-generating process are not quite adequate for the characteristics of the sample. Second, the issue of interdependence clashes directly with the assumption of independence of classical econometrics. Additional research should tackle this contradiction by deriving an appropriate Bayesian estimator for the D-S model, departing from Lindley and Smith's (1972) results on the *exchangeability* of the linear Bayesian model.

Returning to the issues raised in the Introduction of this paper, it would seem that some useful insights could be gained from the placement of a 'filter' between nationallevel changes and their impacts on a specific region. Further empirical analysis and experimentation will have to be performed to help shape the specific form of this filter. During this experimentation, it will be important to incorporate detailed specification of linkages between sectors (through transformations of input-output type relationships) and thus move the modelling system closer to one in which a more complete vision of space-time-sectoral interdependencies is portrayed. However, the system described in this presentation requires, for efficient estimation purposes, that the time dimension exceeds the spatial dimension; the limited reach of the time series will preclude applications of the analysis to a system of all fifty states. For such an application to take place, some form of hierarchical procedure will have to be employed creating additional conceptual and empirical problems about the ways in which spatial dependence are addressed.

REFERENCES

Amos, O.M. (1988) Unbalanced regional growth and regional income inequality in the latter stages of development. *Regional Science and Urban Economics*, 18, pp. 549-566

Amselin, L. (1988) Spatial Econometrics: Methods and Models. Dordrecht: Kluwer.

Bolton, R. (1985) Regional econometric models. *Journal of Regional Science*, 25(4), pp. 495-520.

Chenery, H.B. and Syrquin, M. (1975) Patterns of Development 1950-1970. Oxford

University Press: Oxford.

- Conway, R.S. (1979) The simulation properties of a regional interindustry econometric model. *Papers Regional Science Association*, 43, pp. 45-47.
- Conway, R.S. (1990) The Washington projection and simulation model: A regional interindustry econometric model. *International Regional Science Review*, 13, pp. 141-166.
- Conway, R.S. (1991) An empirical comparison of regional multipliers. In J.H. L. Dewhurst, G.J.D. Hewings and R.C. Jensen (eds), *Regional Input-Output Modeling*. Avebury: Aldershot.
- Cramér, J. S. (1986) *Econometric Applications of Maximum Likelihood Methods*. Cambridge University Press: Cambridge. (1st paperback edition, 1989)
- Dendrinos, D. and Sonis, M. (1990) Chaos and Socio-Spatial Dynamics. Springer-Verlag: New York.
- Henderson, J. and Krueger, A. (1965) *National Growth and Economic Change in the Upper Midwest*. University of Minnesota Press: Minneapolis.
- Johansen, L. (1960) *A Multi-Sectoral Study of Economic Growth*. North Holland: Amsterdam.
- Judge, G.G., Hill, R.C., Griffiths, W.E., Lütkepohl, H. and Lee, T. (1988) Introduction to the Theory and Practice of Econometrics. John Wiley and Sons: New York (2nd. edition).
- Lindley, D. and Smith, A. (1972) Bayes estimates for the linear model. *Journal of the Royal Statistical Society*, (B), 34(3), pp. 112-119.
- Richardson, H. (1978) Regional Economics. University of Illinois Press: Urbana.
- Sonis, M., Gazel, R. and Hewings, G.J.D. (1995) An examination of multi-regional structure: hierarchy, feedbacks and spatial linkages. *Annals of Regional Science* 29, pp. 409-430.
- White, E.N. and Hewings, G.J.D. (1982) Space-time employment modelling: Some results using seemingly unrelated regression estimators. *Journal of Regional Science*, 22(3), pp. 283-301.

APPENDIX

Estimation of the log-linear Dendrinos-Sonis model requires that a log-normal distribution be used. For each region *j*, the joint log-normal density function of the sample of relative population components and the unknown parameters β_j and σ_j^2 can be expressed as a likelihood function of the form:

$$\ln L(\beta_{j},\sigma_{j}^{2}/y_{j},X) = -\left(\frac{T-1}{2}\right)\ln(2\pi) - \left(\frac{T-1}{2}\right)\ln(\sigma_{j}^{2}) - \frac{(y_{j}-X_{\beta_{j}})'(y_{j}-X_{\beta_{j}})}{2\sigma_{j}^{2}} \quad (A.1)$$

Maximization of this likelihood function occurs for the value of the vector β_j that yields the larger probability for the realizations of the observations, y_j . Taking the partial derivative of (A.1) with respect to y_j and setting it equal to zero yields the efficient, consistent and best unbiased maximum likelihood estimator (Judge *et al.*,

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1988, pp. 223-229):

$$\beta_j = [X'X]^{-1}X'y_j \quad \text{with} \quad \beta_j \approx N(\beta_j, \sigma_j^2, [X'X]^{-1}) \tag{A.2}$$

Maximizing (A.1) with respect to σ_j^2 yields the estimator, $\tilde{\sigma}_j^2$:

$$\tilde{\sigma}_{j}^{2} = \sigma_{j}^{2} \frac{[v_{j} - X\beta_{j}]'[v_{j} - X\beta_{j}]}{T - 1} \quad \text{with} \quad E(\tilde{\sigma}_{j}^{2}) = \sigma_{j}^{2} \frac{[(T - 1) - (S + 1)]}{T - 1}$$
(A.3)

which is biased since as $T \rightarrow \infty$, σ^2 does not converge to σ^2 . Multiplication of (A.3 by T/[T - (S+1)] transforms it into σ^2 , i.e., a consistent estimator of σ^2 :

$$\delta_j^2 = \frac{(y_j - X\beta_j)'(y_j - X\beta_j)}{(T-1) - (S+1)}$$
(A.4)