

## **AN ANALYSIS OF THE INTERDEPENDENCE OF REGIONAL ECONOMIES**

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**ABSTRACT** Multiregional computable general equilibrium (MRCGE) models are beginning to become established as an important tool in regional economic analysis. In Australia, two large-scale MRCGE models, FEDERAL and MONASH-MRF, are in widespread use. A great advantage of these models for analysing policy-oriented regional questions is their capacity to handle enormous detail about the structure of the regional economies concerned and about the interrelationships which exist between them. On the other hand, their very complexity exacerbates the task of uncovering the major economic mechanisms which lie behind a set of simulation results. In this paper we use a miniature version of the two-region FEDERAL model, MF, as a means of simplifying the task of establishing the major mechanisms involved in the interregional transmission of economic impacts. We look at the illustrative case of a payroll-tax cut by one of the model's two regional governments. Central to our analysis is an extension of the conceptual framework of international trade theory to take into account the greater mobility of factors that exists across regions than across nations. This enables the impact on the real consumption of the passive region to be decomposed into three effects: a resource-allocation effect, a resource-movement effect and a terms-of-trade effect. We then demonstrate that for the MF payroll-tax cut simulation, the transmission of impacts from the policy-initiating region to the passive region can be easily explained in terms of the three effects.

### **1. INTRODUCTION**

Multiregional computable general equilibrium (CGE) models have become a well-established analytical tool over the present decade. Examples of applications of such models can be found, for instance, in Jones and Whalley (1989) and Hewings and Gazel (1992). In Australia, multiregional CGE models have now been used to tackle a wide variety of major regional economic questions (see for instance, Dixon, Madden and Peter, 1990, 1993; Madden, 1993; Madden, 1995; Naqvi and Peter, 1996; Wittwer and Bright, 1997; Giesecke and Madden, 1997; Crowe, 1996; and NSW Treasury/CREA, 1997).

A feature of the two major Australian multiregional CGE models, FEDERAL

and MONASH-MRF, are their very large size.<sup>1</sup> The first of these models depicts up to 130 industries<sup>2</sup> in two regions, including up to eight industries supplying margin services on intraregional, interregional and international trade. FEDERAL also contains very detailed modelling of the finances of two tiers of government, with a range of explicitly-modelled regional (state) and federal government taxes affecting the purchase price of commodities, and of regional income. The MONASH-MRF is an eight-region model, containing 13 industries. While not as many regional taxes are explicitly modelled as in FEDERAL, it nevertheless contains very detailed government accounts for both tiers of government. It also contains capital and (external) debt accumulation equations, together with demographic equations, which allow for annual-average forecasts to be undertaken with the model in addition to its comparative-static capabilities.

Extensions to both models are continually being made. For instance, Giesecke (1997a) has added sufficient equations to allow FEDERAL to be used to undertake year-on-year forecasts, as well as historical simulations that allow the analysis of structural change (Giesecke, 1997b).

The advantage of using models of such size is their ability to handle enormous detail about the structure of regional economies. This is particularly important in simulations concerning questions where regional and industry detailed results are of pre-eminent interest. However, large-scale modelling carries considerable costs as well, particularly the large amount of painstaking work that must be undertaken to ensure that key results can be explained in terms of model mechanisms.

In the course of undertaking many policy-oriented simulations with large-scale multiregional models, we have become interested in analysing the principal underlying causes of movements in real regional aggregates. At the national level, the effects of alternative macroeconomic closures on the major components have been very well examined (see Adams and Parmenter, 1994). However, the much more mobile nature of factors at the regional level complicates this sort of analysis at the sub-national level.

In order to start the pursuit of this line of inquiry, we employ in this paper a small two-region model for a hypothetical economy. We take as our starting point a miniature multiregional model developed by Madden (1993). This model, entitled MINIFEDERAL (or MF for short), was developed principally as an expository model. It contains some of the interesting regional features of FEDERAL. In the next section we briefly outline MF. However, some features of MF still contain complications that are a hindrance to the analysis proposed. In the subsequent section we deal with certain amendments that enhance the usefulness of the model for our purposes.

While the large models are very flexible analytical tools, the miniature model

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<sup>1</sup> FEDERAL is outlined in Madden (1996) and fully described in Madden (1990). Monash-MRF is outlined in Naqvi and Peter (1996) and fully described in Peter *et al.* (1996).

<sup>2</sup> Most versions of the model comprise about 40 to 60 industries in each region. FEDERAL (NSW), the largest version of the model, covers 130 industries in each of the two regions (NSW and Rest of Australia).

used here facilitates our analytical research both in easing the task of interpreting results, and in allowing the quick computation of regional effects for illustrative shocks under a variety of scenarios. In particular this affords easy analysis of a much wider range of alternative closures and data-base changes (reflecting different regional structures) than is practicable with larger models.

## 2. ANALYTICAL FRAMEWORK

The question which we wish to examine is "what are the key mechanisms by which a shock to one region transmits effects to other regions within the nation?" We feel this is an important question, because a general answer to it should prove of considerable value to multiregional modellers in explaining results from many simulations.

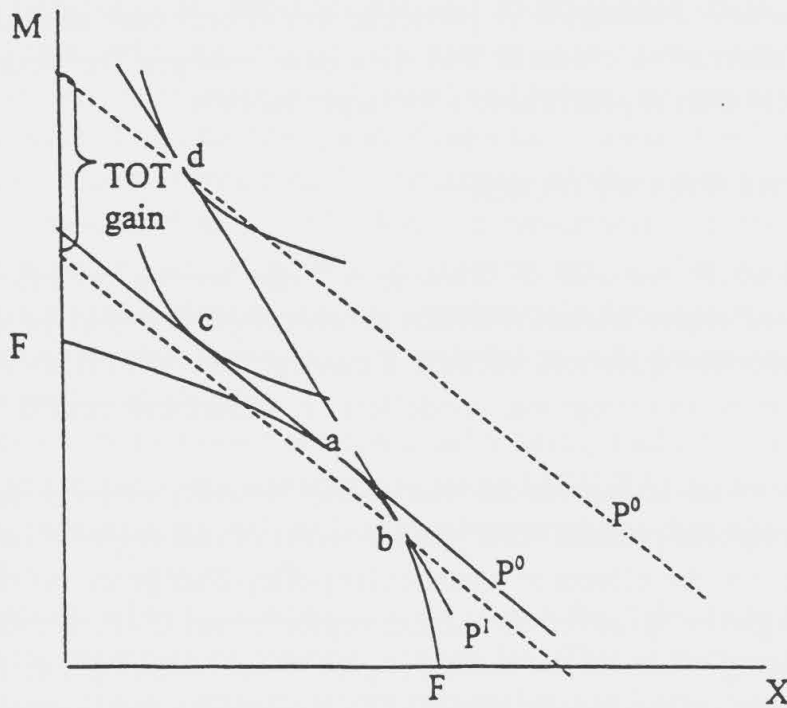
We particularly wish to analyse the relationship between interregional spillovers in terms of gross regional product (GRP) and in terms of real regional consumption. It is often the case that the effects of a particular policy change in one region result in a significant negative spillover on another region's real GRP, without having a particularly damaging effect on the second region's real consumption. It may even happen that the effect on the second region is to increase its real consumption. It is important to explain how such an initially counter-intuitive result can occur. Examination of this question can also throw light on whether the same mechanism might nullify the positive effects that the policy-induced shock might otherwise have had on the initiating region's welfare, and whether this contrary effect might be mitigated or not.<sup>3</sup>

In order to develop an analytical framework which can keep track of the transmission of interregional spillover effects we commence with a well-known technique of international trade theory. We note that the fundamental difference between national and regional economies is the degree to which factors are mobile. Factors can be considered to be perfectly mobile between the regions of a nation, while being far less mobile between nations. Hence, we will allow this feature of factor mobility to enter into the conventional model of trade theory in order to develop an explanation of how economic effects are fundamentally transmitted across regions.

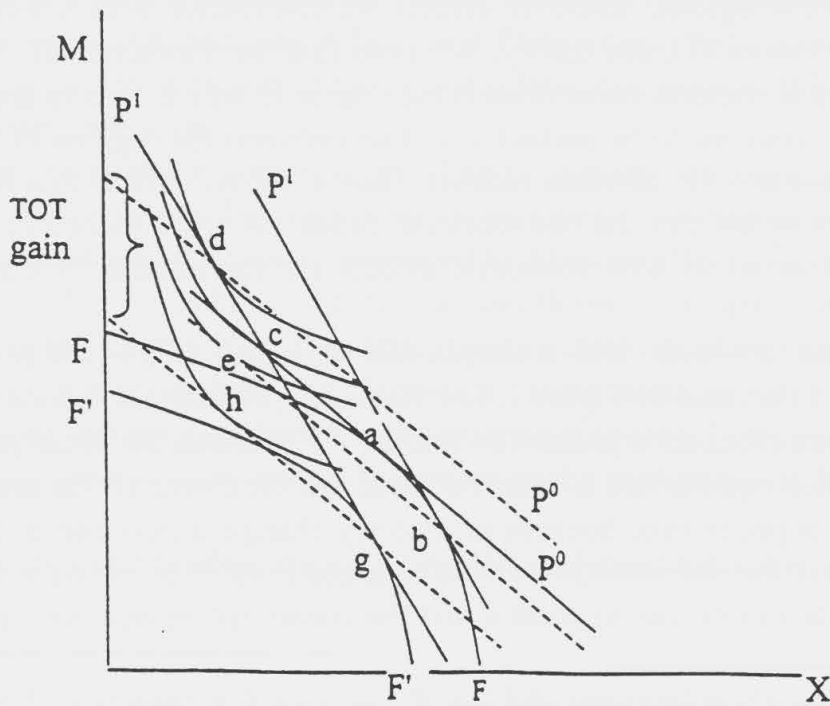
We commence, however, with a simple case in which factors are not mobile across regions, as illustrated by Figure 1. Let the region produce two goods  $X$  and  $M$  according to a given production possibility frontier  $FF$ . Assume the initial price ratio is  $P^0$ . The production equilibrium is thus at point  $a$ , and the consumption equilibrium at point  $c$ . Let us suppose that, because of a policy change introduced in the other region (not shown), the domestic price ratio changes from  $P^0$  to  $P^1$ . As a result the production equilibrium moves to point  $b$  and the consumption equilibrium moves

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<sup>3</sup> This is an important matter when considering the efficacy of regional government policy instruments. For instance, Giesecke and Madden (1997) found some divergence between the rankings of regional-stimulating instruments in regard to their effect on real gross state product and real consumption. The divergence was greatly increased when the effect on real per-capita consumption was considered.



**Figure 1.** Transmission of Effects Across Regions When Factors Are Not Mobile Across Regions.



**Figure 2.** Transmission of Effects Across Regions When Factors Are Interregionally Mobile.



to point  $d$ . Compared to point  $a$ , the constant price GRP of the region at point  $b$  falls, as shown by the dotted line passing through  $b$ . Hence the "real" GRP of the region falls as a result of some exogenous (policy) change in the other region. Nevertheless, the consumers in the region can enjoy a higher living standard as shown by the position of the dotted line passing through the point  $d$  (the new consumption equilibrium). The gap between the two dotted lines is the gain in the terms of trade enjoyed by the region under study. The policy change in the other region caused that region's export prices to fall, while the relative price of the exports of our region rises. Hence, in this simple model, the effects are transmitted via two effects: changes in the real GSP (which is negative) and changes in the terms of trade (which is favourable).

Now let us consider a case, illustrated in Figure 2, in which factors are interregionally mobile. In this case factors will move from one region to the other region in pursuit of higher factor prices. If a region introduces an expansionary policy (one that makes output expansion more profitable at the going prices), this will not only cause the interregional terms of trade to change, but will also alter factor prices - for example wages will rise. This will in turn cause labour to move from the other region to support the expansion. The other region, however, will experience a contraction in its factor supply and its production possibility frontier will shrink, as shown by the curve  $F'F'$  in Figure 2.

As in Figure 1 we start from production equilibrium at point  $a$  and consumption equilibrium at point  $c$ , with the price ratio given by  $P^0$ . As a policy shift in the policy-initiating, or "active", region causes the price ratio to move to  $P^1$ , the production equilibrium moves to point  $b$ , and the "real" value of the output would support a consumption equilibrium at  $e$ , which is lower than point  $c$ . The expansionary policy in the active region causes factor prices to rise, and as a result some factors leave the "slack" (or passive) region and move to the active region. This causes the production possibility frontier in the slack region to shrink to  $F'F'$ . Assuming that the relative price is still given by  $P^1$ , the "real" value of this output level supports a consumption equilibrium at point  $h$ . Since, at price  $P^1$  the regions trade and the slack region benefits from the higher price for its exports, the final consumption equilibrium is sustained at point  $d$ . The real income difference between the points  $d$  and  $h$  is the terms of trade effect.

Thus, the slack region, which is affected by the change that has taken place in the other region, has moved from point  $c$  to point  $d$  as its consumption equilibrium. This change is desirable. At the same time its GRP measured at constant price has fallen as shown by the gap between the dotted line passing through the point  $g$  and the initial price line  $P^0$ , passing through the point  $a$ .

A fall in real GRP and a rise in real consumption resulting from the inter-regional spill-over effect can be consistent with the economic logic. The inter-regional spill-over, thus, can be summarized in terms of three distinct effects: a resource allocation effect (move from  $c$  to  $e$ ); a resource movement effect (move from point  $e$  to  $h$ ) and a terms of trade effect (move from point  $h$  to point  $d$ ). If a positive terms of trade effect dominates the negative reallocation effect and resource movement effects then the slack region may benefit from the changes taking place

in the active region. On the other hand, however, if the resource movement effect dominated the positive terms of trade effect (or even more so if the terms of trade changes become unfavourable) a slack region will be destined to lose from the changes taking place in other "active" regions.

We will use this classification of the above three effects to decompose the results of an illustrative regional policy shock, a reduction in regional payroll taxes. However before doing so we discuss the main features of the small multiregional CGE model that we employ to simulate the effects of the shock.

### **3. THE MULTIREGIONAL CGE MODEL USED: MINIFEDERAL**

#### **3.1 Relationship to FEDERAL**

The construction of MF involved simplifying FEDERAL by the deletion of the modelling of regional industry investment, technical change, most forms of taxes and categories of government expenditure, and margins. Only one type of labour is modelled in MF and income taxes are levied only on labour inputs. While the model does recognise interregional (and foreign) ownership of capital, it only models fixed capital, thus omitting working capital. Further simplification is achieved by allowing for only three industries, none of which are multiproduct. Substitution possibilities in both production and consumption are specified in a more simplistic (and restrictive) way than in FEDERAL.

Despite all of these omissions MF provides a valuable introduction to the FEDERAL model. The absence of detail allows for a ready overview of the model's structure. In particular MF helps makes clear to those new to CGE modelling how the model's equations are derived using standard producer and consumer theory, etc., and how the model's coefficients are derived from an interregional input-output table.

#### **3.2 The Multiregional Input-Output Data Base**

The forerunner to the CGE model was the input-output model, still widely used in regional analysis and a tool familiar to most regional economists. The primary data source for CGE models is still input-output data and a description of this data base for MF forms a useful starting point for our description of MF.

The input-output information required for MF is set out in Table 1 which shows an absorption matrix for a fictitious country. Madden (1993) chose the numbers in the table to conform with certain stylised facts regarding certain Australian regions.

It is worth briefly examining the structure of the multiregional input-output absorption matrix depicted in Table 1. While the headings are fairly standard, one immediately noticeable omission is investment from the categories of final demand. If we were to compare the MF input-output data base with that of FEDERAL, the schematic representation of which is shown in Figure 3.1 of Madden (1990), we would see that a substantial amount of simplification comes from the absence in MF of sales tax and various other tax categories, plus the omission of any explicit

Table 1. Absorption Matrix for MINIFEDERAL

	Intermediate Users						Final Demands				Row Totals		
	Region 1 Industries		Region 2 Industries		Household Consumption		Exports Abroad		Government Expenditure			-Duty	
	1	2	3	1	2	3	1	2	Federal Govt	Region 1 Govt			Region 2 Govt
Region 1	1	4	9	2	3	0	3	0	6	0	0	0	27
	2	1	10	0	22	10	12	2	12	0	0	0	73
	3	5	7	22	0	0	50	0	0	6	19	0	109
Region 2	1	1	4	8	16	4	1	20	2	0	0	0	57
	2	4	15	9	50	78	4	61	20	0	0	0	256
	3	0	0	10	65	102	0	225	0	53	0	51	506
Imports from Abroad	1	0	0	0	0	0	0	1	0	0	0	0	1
Labour	2	1	4	0	5	20	5	20	0	0	0	0	50
	3	0	0	0	0	1	0	0	0	0	0	0	0
Post-tax	5	9	28	9	29	128	128	128	128	0	0	0	0
Income Tax	2	4	12	4	13	57	57	57	57	0	0	0	0
Payroll Tax	0	1	2	0	2	8	8	8	8	0	0	0	0
Capital	1	5	4	19	3	2	2	2	2	0	0	0	0
	2	0	6	11	25	100	100	100	100	0	0	0	0
	3	0	2	1	8	5	5	5	5	0	0	0	0
Federal Govt Prodn Tax	0	1	2	0	3	9	9	9	9	0	0	0	0
Regional Govt Subsidies	-1	-1	0	-1	-3	0	0	0	0	0	0	0	0
Column Totals	27	73	109	57	256	506	506	506	506	53	19	51	506

representation of markup commodities such as the use of transport services and retail trade to facilitate the direct flow of commodities. Other complexities omitted from the MF data base are land inputs, a disaggregation of labour inputs and working capital.

It can be seen, looking at the export column, that MF excludes the possibility of commodities imported from abroad being subsequently exported without any domestic processing and therefore the associated vector (imports from abroad to exports abroad) is null. Although MF allows for the theoretical possibility of purchases of imports from abroad by governments, we see that zero purchases of this type is assumed in the fictitious data base. It should be noted that, the major way the import sales rows differ from the domestic commodity sales rows is an entry showing the negative of the duty paid on imports from abroad. If one adds right across a row of import sales the landed-duty-free (ldf) value of the imported good (in the domestic currency) is obtained.

While most headings are self-explanatory it needs to be pointed out that the three rows of capital inputs distinguish the three regions (region 1, region 2 and foreign) of capital ownership. Note that federal government production taxes (less subsidies) are always positive in the base-year. On the other hand regional governments only subsidise industries (net taxes are negative for all regions in the data-base year).

In Table 2 we show GDP aggregates which are derived from the Table 1 multiregional input-output table.

It can be seen from Table 2 that Region 2 makes up the bulk (around four-fifths) of the national economy. Assume for the sake of exposition, that the three industries in each region are: Primary, Secondary and (non-traded) Tertiary. It can be seen from Table 1 that Region 1 has a significantly lower tertiary-orientation than Region 2, and is, in particular, far more primary-sector intensive. Region 1 is the poorer region,<sup>4</sup> is more export (abroad) oriented, but while enjoying a surplus balance of trade with other countries it has a deficit on its balance of trade with Region 2.

Very little data is required by the model other than that which can be found in Table 1. Production and utility functions underlying MF are such that the only extra parameter information required relates to foreign export demand elasticities. A small amount of additional labour-force information is required, while the required government accounts items (shown in Table 3) can all be derived from Table 1, with the exception of unemployment benefits and interregional government grants.

### 3.3 Theoretical Structure of MF

#### 3.3.1 Johansen Approach

Like all Australian CGE models, MF is of the Johansen class. A comprehensive examination of the Johansen approach can be found in Chapter 3 of Dixon *et al.* (1992). The most characteristic feature of this class of model concerns the solution

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<sup>4</sup> Based on a lower pre-tax residential income per person in the labour force. Assuming similar participation rates and age structures this translates into a lower income per head.

**Table 2.** GDP Accounts for Fictitious Economy

	Region 1	Region 2	National
Household Consumption	75	329	404
Investment	0	0	0
Government Expenditure	25	104	129
Overseas Exports	20	22	42
Overseas Imports	9	42	51
Interregion Exports	39	45	84
Interregion Imports	45	39	84
GDP (Expenditure)	105	419	524
Labour	60	240	300
Capital	40	156	196
GDP (Factor Cost)	100	396	496
Indirect Taxes	5	23	28
GDP (Income)	105	419	524

**Table 3.** Governments Accounts Data for MF

<b>Federal Government</b>			
<u>Receipts</u>		<u>Outlays</u>	
Income Tax	92	Current Outlays	59
Import Duties	6	Unemployment Benefits	9
Production Taxes	15	Grants to Regional Governments	63
Total	113		131
<b>Regional Governments</b>		Region 1 Govt	Region 2 Govt
<u>Receipts</u>			
Payroll Taxes		3	10
Grants from Federal Government		18	45
Total		21	55
<u>Outlays</u>			
Current Outlays		19	51
Production Subsidies		2	4
Total		21	55

method they invariably employ. This involves the transformation of the model's equations from a set of non-linear relationships in the levels of the variables into a set of linear relationships, usually in percentage rates of change of the variables.<sup>5</sup>

Consider a general equilibrium model represented as a system of  $m$  non-linear equations,

<sup>5</sup> Alternative Johansen solution methods involve expressing the variables in terms of changes in, or changes in the logarithms of, the levels of the variables. It should be noted that FEDERAL could be solved by methods other than Johansen's. However, as we shall see, the Johansen solution method is integral to the way the MF model is written.



$$F(Z) = 0 \quad (1)$$

where  $Z$  is a vector of length  $n$  with values that satisfy equilibrium and  $n > m$ . With the Johansen solution method (1) is converted via logarithmic differentiation of each equation to,

$$A(Z)z = 0 \quad (2)$$

where  $A(Z)$  is an  $m \times n$  matrix of coefficients whose components are a function of the elements of  $Z$  and  $z$  is an  $n \times 1$  vector of percentage changes of the elements of  $Z$ . Johansen solution of (2) makes use of an initial solution,  $Z^1$ , which in the case of MF is obtainable from the data base which was outlined in section (2.2). The replacement in (2) of  $A(Z)$  by the fixed matrix,  $A(Z^1)$  means that (2) is only properly representative of (1) for small changes in the elements of  $z$ .

The fact that (2) is only locally exact meant,<sup>6</sup> for many years, that CGE modellers were faced with a trade-off between the considerable computational advantages of the Johansen method and its inherent linearisation errors. However, new computer software packages (see, for example, Hertel *et al.* (1992)), have brought an end to the problem of approximate solutions with linearised models, via the use of multi-step computational procedures.<sup>7</sup>

Readers interested in a comprehensive discussion of the methods for eliminating Johansen's linearisation errors should consult Dixon *et al.* (1992) and Pearson (1991). For the remainder of this paper we shall think in terms of a one-step solution (the Johansen solution) to the MF model. This simplifies the exposition and does no harm since invoking more accurate solution methods can be viewed as a run-time choice not involving any difference in approach for the CGE modeller than that for the one-step solution.

Apart from their equations being written in linear form, Johansen models have much in common with most other CGE models. A typical theoretical structure of a Johansen model can be put forward in terms of broad groups of equations which one would expect to find in any model. Following Dixon *et al.* (1992) we list five groups:

1. household and other final demands for goods;
2. input-demand equations - for primary factors and intermediate goods;
3. price equations relating the prices of goods to their costs;
4. market-clearing equations - for primary factors and goods;
5. miscellaneous equations - usually definitional equations computing aggregates such as GDP or a consumer price index.

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<sup>6</sup> Exactness requires perturbations from  $Z^1$  to be infinitesimal, but applied economics has not reached a level of exactness (in theory or parameter estimation) where small linearisation errors associated with small shocks are not acceptable for most policy analysis.

<sup>7</sup> The simplest method is Euler's which involves decomposition of the shocks to the exogenous variables into a number of equal parts. This allows the  $A(Z)$  matrix to be updated after the solution for each part shock is computed. The solution for subsequent part shocks makes use of the most recently updated  $A(Z)$  matrix.

In his description of MF Madden basically adheres to this classification, at least in so far as the first four groups are concerned. For MF the fifth group consists of the definition of some aggregate variables, indexation equations and the modelling of government accounts. An important part of the fifth group concerns the calculation of regional income.

In this paper we provide only a brief verbal explanation of MF's theoretical specification. However, the model's equations are fully depicted in Appendix A of this paper, while the variables are defined in Appendix B and the model's coefficients and parameters are described in Appendix C. It is important to highlight the convention adopted throughout the rest of this paper (and already employed in equation (2)) is that, where variables are concerned **upper case** letters refer to the *level* of the variables, while **lower case** variables refer to *percentage* changes in the variables.

### 3.3.2 Equation System

#### *Final Demands*

A representative household in each region maximises a utility function subject to an aggregate expenditure constraint-determined by after-tax factor income, transfers and savings. The utility function is a nested one, the form of which is shown diagrammatically in Figure 3. It can be seen at the top level that regional households treat effective units of goods as non-substitutes, while at the second level commodity *i* from the three different sources (home region, other domestic region and foreign) are imperfect substitutes.<sup>8</sup>

Substitution of the representative regional consumer's problem yields (after linearisation) equation (1) of Appendix A. Madden (1993) provides a full description of the computation of the price elasticities from the data base.<sup>9</sup> Note that the utility function is homothetic and thus all expenditure elasticities are equal to unity.

The quantity of foreign demand for exports from regional industries is modelled in MF as being dependent on the foreign-currency supply price, yielding equation (2) of Appendix A.

The level of each government's demand for current commodities is effectively modelled to depend only on policy-shift parameters (equations (3) and (4) of Appendix A).

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<sup>8</sup> This form of utility function was used in Dixon *et al.* (1982) for the development of Miniature ORANI. The households demand section of MF is closely based on the Miniature ORANI theory.

<sup>9</sup> The computations made use of the Hicks-Slutsky partitioning of the total price elasticity effect into substitution and income effects. Compensated price elasticities between different types of goods are zero. The remaining compensated demand elasticities are solved for by minimising the total cost of purchasing good *i*. It turns out the solution allows the required compensated elasticities to be computed from cost shares. It is then straightforward to compute the uncompensated price elasticities.

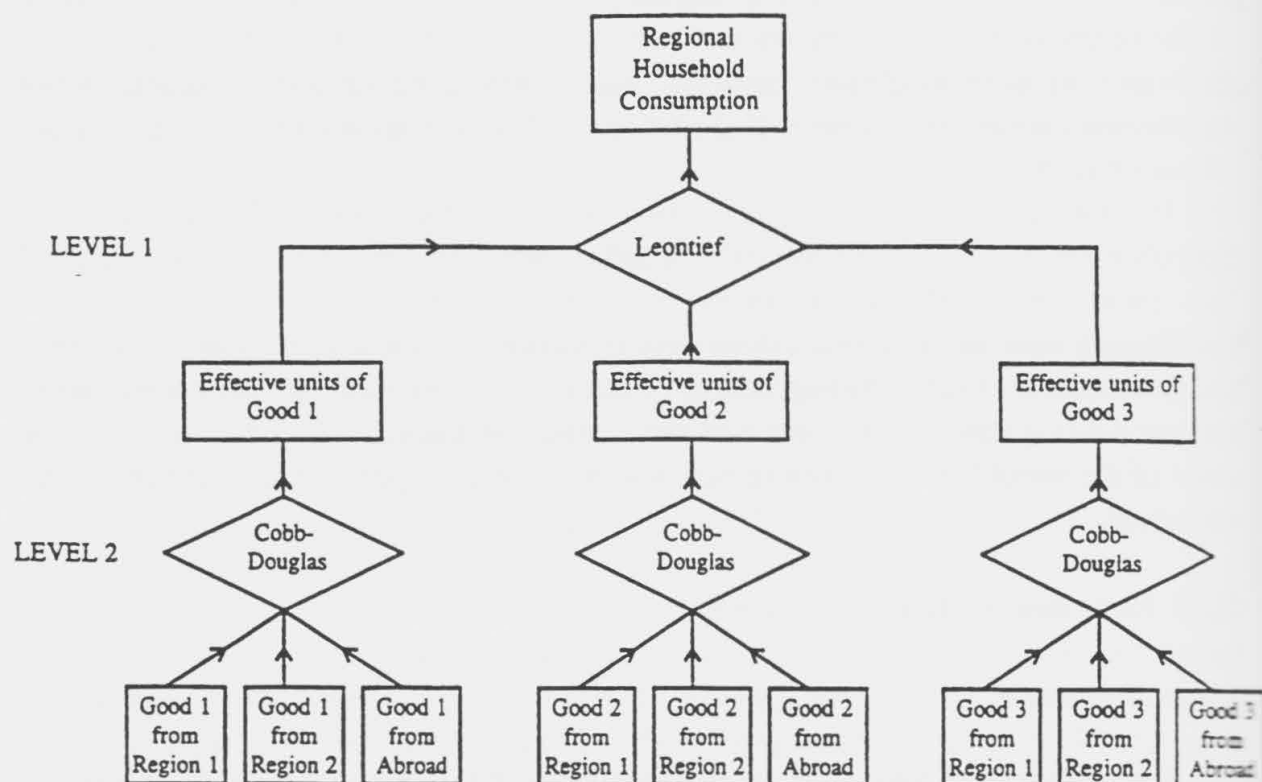


Figure 3. Nesting Structure for Regional Household Consumption MF

### Input Demands

Each producer in a regional industry chooses its inputs to minimise the cost of producing any given output level subject to a constant returns-to-scale production function. Producers are assumed to be price-takers. As with most CGE models, production functions are assumed to be weakly separable. They take the form shown in Figure 4. At the top level there are effective inputs of produced commodities and effective inputs of a primary-factor composite that are required in fixed proportions.<sup>10</sup> Nested into the top-level Leontief production function are a number of second-level non-linear functions which allow substitutability between sub-types of inputs within the broader input types. The produced inputs can be sourced from three geographical areas (region 1, region 2 and foreign) according to a Cobb-Douglas function.<sup>11</sup> Similarly effective units of primary factors are a Cobb-Douglas combination of effective units of labour and region-industry-specific capital.

The solution of the producer's problem gives rise to standard (linearised) input demand equations shown as equations (5) and (6) of Appendix A. It can be seen that a producer's input demands are functions of the required industry's activity level and

<sup>10</sup> The term *effective* input of a particular type refers simply to any combination of sub-types of that input which provide a given level of productivity capacity from that input.

<sup>11</sup> FEDERAL employs a CRESH (constant ratio of elasticities of substitution homothetic) aggregation of sources.

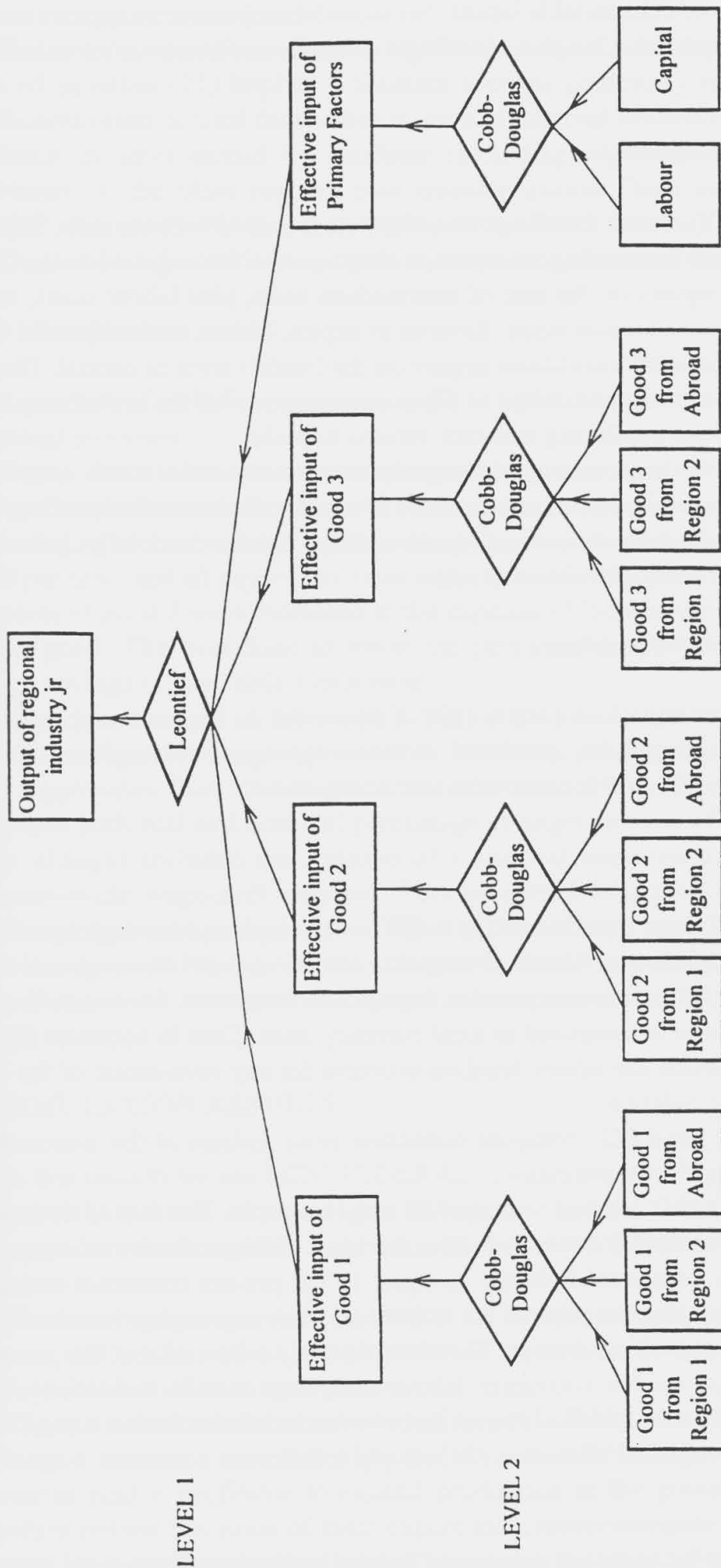


Figure 4. MF Production Technology

of relative prices of substitutable inputs. No substitution parameter appears in either equation, since the Cobb-Douglas specification implies an elasticity of substitution of unity.

### *Zero Pure Profit Equations*

Equations (7) to (9) ensure that the prices of goods are equal to their costs. Equation (7) is derived from linearising an equation that equates the value of output to the total payment of inputs (i.e. the sum of intermediate costs, plus labour costs, returns to capital and net production taxes. Returns to capital covers normal profits which vary as differing industry conditions impact on the (rental) price of capital. The non-appearance of any quantity variables in (7) is a consequence of the underlying nested production functions exhibiting constant returns to scale.

Equation (8) is the percentage-change form of an equation which ensures the domestic-sale price of imported commodities is equal to the cost of importing them. Similarly (9) is a linearisation of an equation which relates the foreign price of an export good to its domestic currency price.

### *Marketing-Clearing Equations*

These appear as equations (10) to (12) of Appendix A. Demand and supply are equated for each commodity produced in each region and for regional-industry capital. Aggregate demand for labour is also computed.

### *Other Equations*

The first set of these equations, (13) to (20), cover various trade aggregates. The equations are straightforward and cover both international and interregional trade. The national trade balance is computed in foreign currency units. However, the total regional trade balance is computed in local currency units. Thus in equation (20) we find the last term inside the square brackets accounts for any revaluation of the base-year national trade balance.

Equations (21) and (22) compute consumer price indices at the national and regional levels respectively.

Equations (23) and (24) deal with the MF wage structure. The first of these is the linearised form of an equation which defines the level of the producer real-wage cost per labour unit in a regional industry as equal to the pre-tax consumer wage per region  $r$  labour unit plus the payroll tax imposed by the regional government on a unit of labour in regional industry  $jr$ . The absence of a  $j$  subscript, but the presence of an  $r$  superscript, on the consumer labour-unit wage rate is indicative of the assumption that labour is (perfectly) mobile between industries within a region but immobile between regions.<sup>12</sup> Equation (24) simply indexes the consumer wage to the

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<sup>12</sup> This does not prevent assuming variations in regional employment levels in the short run. Also, see 3.3.3 for amendment to allow interregional labour mobility.



national consumer price index.

The next four equations compute regional household consumption. Note that the form of equation (25) implies a constant average propensity to consume. Share coefficients relate to total disposable income of regional households. This includes all factor incomes earned by residents (including capital income earned from investment in the other region), plus transfer income (here just unemployment benefits), less all direct taxes (here just taxes on labour income).

The remaining equations compute the accounts of each of the governments.

### **3.3.3 Modifications to MF**

A number of modifications were made to MF to enhance its suitability for our analytical purposes.

Firstly, the data base was altered, particularly by removing certain complications to the distribution of income. All interregional and foreign ownership was eliminated, all duties were set equal to zero, all income tax rates were set uniformly (at 40 per cent) and all payroll tax rates were set uniformly (at 15 per cent). Region 2 exports of good 2 were increased at the expense of local household consumption of that good. This was done to lower the persistent dependency of region 2 on foreign savings in the Table 1 data base.

The main changes to the model's theory were to introduce equations which modelled national and regional accounts. Equations dealing with each of the main GDP aggregates from the expenditure side were added. The amended model computes both real and nominal percentage changes in each of these aggregates. Other changes included the addition of a national employment variable and an economy-wide wage-shift variable. This permitted labour to be interregionally mobile while holding employment fixed at the national level. Finally we added equations to model terms of trade changes. It should be noted that these terms of trade changes are measured as changes in real income units (not in percentage change terms).

## **4. SIMULATION RESULTS**

In this section we use MINI FEDERAL to examine the effects of a 10 per cent reduction in the rate of payroll tax in region 1 on real GRP and real regional consumption in both region 1 and region 2, and similarly for a region 2 payroll-tax reduction.

The list of exogenous variables used in the simulation is set out in Table 4. It can be seen that while national employment is treated as exogenous each regional employment is treated as endogenous.

The main results of the simulation are set out in Table 5. Looking at the first row of Table 5, it can be seen that when the tax reduction is initiated by region 1, its industries find it profitable to expand production at the going prices. They can therefore reduce the price of their export and import-competing commodities in order to increase their level of sales. This causes the terms of trade to change and

Table 4. List of Exogenous Variables

Variable	Number	Description
$f^{(X)}$ $f^{(ir)}$	6	Shift variable for foreign export demands.
$c_R$	1	[Note, this variable not modelled.]
$f^{(FG)}$	1	Shift term for federal government expenditure.
$x^{(RG)r}$	2	Aggregate region $r$ government expenditure.
$p^{(CT)jr}$	6	Federal government production taxes.
$p^{(ST)jr}$	6	Regional government industry subsidies.
$p^{(F)}$ $p^{(i3)}$	3	C.i.f. foreign currency import prices.
$t_i$	3	Ad valorem import duties.
$\phi$	1	Exchange rate.
$k_j^r$	6	Regional industry capital stock.
$l$	1	National employment <sup>#</sup>
$f_{jr}^r$ $f_{jr}^{(F1)}$	2	Regional wage shift variable.
$t_{(PR)}$	6	Payroll tax per labour unit.
$f^r$	2	Region $r$ labour force.
$\Delta B_{(FG)}$	1	Federal government borrowing requirement.
$\Delta B_{(RG)}$	2	Regional government borrowing requirement.

#: Variable not in original version of MF.

interregional spillover effects begin to arise. Region 1's GRP at constant prices increases by 0.09 per cent, but as predicted the constant price GRP of region 2 is projected to fall by 0.02 per cent. Region 1 loses from the terms of trade change (this includes international terms of trade changes as well), while region 2 gains from the terms of trade change.<sup>13</sup>

The real GRP and terms of trade results given in Table 5 can be converted into contributions to the percentage change in regional real consumption. In turn this allows us to perform the decomposition of effects discussed in section 2. This decomposition is shown in Table 6. It should be noted that given the list of exogenous variables set out in Table 4, we cannot use the Table 5 GRP figure to compute the resource allocation effect ( $a$  to  $b$  in Figure 2). The Table 5 GRP figure gives the movement from point  $a$  to point  $g$ , and thus includes some interregional resource movements. In generating Table 6 we overcame this difficulty by recomputing the GRP result with a list of exogenous variables characterised by interregional immobile labour. The residual between the two scenarios is reallocated to the resource movement component, which also includes some second order effects.

<sup>13</sup> It should be kept in mind (as explained in the previous section) that terms of trade changes are measured as changes in real income units, while all other variables are reported in percentage change terms.

**Table 5.** Interregional Effects of a Payroll Tax Reduction in a Region

	Effects of Payroll Tax Reduction on							
	Real GRP		Terms of Trade		Real Consumption		National	
	Region 1	Region 2	Region 1	Region 2	Region 1	Region 2	Wage Rate	CPI
Payroll Tax Reduced in Region 1	0.09	-0.02	-0.01	0.23	0.25	0.07	0.62	0.46
Payroll Tax Reduced in Region 2	-0.46	0.12	0.38	0.65	-0.20	0.63	2.63	1.98

**Table 6.** Decomposition of the Interregional Effects on Regional Real Consumption of Payroll Tax Reduction in a Region

	Transmission of the Effects of Payroll Tax Reduction via							
	Resource Allocation		Terms of Trade		Resource Movement		Real Consumption	
	Region 1	Region 2	Region 1	Region 2	Region 1	Region 2	Region 1	Region 2
Payroll Tax Reduced in Region 1	0.01	0.00	-0.01	0.07	0.25	-0.01	0.25	0.07
Payroll Tax Reduced in Region 2	-0.04	0.01	0.50	0.21	-0.67	0.41	-0.20	0.63

From the first row of Table 5 it can be seen that when payroll tax is reduced in region 1, real consumption rises by 0.07 per cent in region 2 despite a *fall* of 0.02 per cent in real GRP. The first row of Table 6 provides the explanation. Real consumption rises in the face of the fall in real GRP because of a contribution of 0.07 per cent from the terms of trade effect, with the small contributions of the resource-allocation and resource-movement effects partly cancelling out.

From the second row of Table 5 it can be seen that when payroll tax is reduced in region 2, real consumption falls by only 0.2 per cent in region 1, despite a 0.46 per cent fall in real GRP. Again Table 6 provides an explanation. From the second row of Table 6 we see that a fall in region 1 real consumption is cushioned by a 0.5 per cent contribution from the terms of trade effect which substantially offsets the -0.67 per cent contribution from the resource movement effect.<sup>14</sup>

## 5. CONCLUDING REMARKS

In Section 2, we used a well-known technique from the theory of international trade to develop a decomposition of the effects of a policy shock in one region on the real consumption of another region. The decomposition was three-fold and consisted of: a resource-allocation effect, a resource movement effect and a terms of trade effect.

In section 4 we returned to this decomposition. We began by using a miniature two-region model, MF, to simulate the effect of a policy shock in region 1 on macro variables in both regions and similarly for a region 2 shock. The policy shock concerned was a 10 per cent reduction in the rate of payroll tax. The results of this simulation included the effects of the reduction in the rate of payroll tax in region 1 on the real consumption of region 2 and vice versa. Using other results from the simulations we were able to decompose these real consumption effects, in both cases, into the resource-allocation, resource-movement and terms-of-trade components.

With these decompositions at our disposal we were then able to rationalise one puzzling feature of the simulation results, - the contrasting behaviour of real GRP and real consumption. What the simulation results showed was that while real GRP *fell* by 0.02 per cent in region 2 in response to the payroll tax reduction in region 1, real consumption *rose* by 0.07 per cent. The decomposition results enabled us to

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<sup>14</sup> It will be seen from row 2 of Table 5 that the terms of trade improve for both regions when the payroll tax policy is initiated by region 2, the larger of the two regional economies. Recall that the terms of trade shown in Table 5 are for total trade by a region (interregional and international). While region 2 suffers a slight diminution in its interregional terms of trade, it also experiences a considerable improvement in its international terms of trade, noticeably larger than region 1's terms of trade improvement. This explains the relative size of the regions' terms of trade effects for the region 2 policy. That there is a projected increase in the international terms of trade springs from our choice of exogenous variables in Table 4. The international trade balance is left endogenous. With the nation's trade balance deteriorating and real wage increasing by 0.8 per cent, domestic price levels rise, increasing the international terms of trade.



"explain" this counter-intuitive result in terms of an improvement in region 2's terms of trade.

Similarly the simulation results showed that while real GRP fell by nearly half a per cent in region 1 in response to the payroll tax reduction in region 2, real consumption fell by only one fifth of a per cent. Again the decomposition results allowed us to advance a logic for this outcome. Here, too, the logic ran in terms of a substantial improvement in region 1's terms of trade giving support to real consumption in region 1 in the face of the unfavourable resource movement for which the large decline in region 1's GRP was mainly responsible.

The outcome of the simulation exercise which we have undertaken with MF in this paper, as summarised above, would seem to give considerable support to our view that the decomposition method is an effective tool for analysing and understanding the *interdependence* of regional economies. Analysis of the extent to which a region might be positively or negatively affected by policy changes initiated by another region is an essential prior step to considering the scope for cooperative behaviour between regional jurisdictions.

We further believe that this paper has demonstrated that the results generated by a miniature CGE model, can often throw valuable light on the results generated by large-scale working models of the CGE class, simply because the former are, by comparison, easy to rationalise or explain.

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**APPENDIX A. MINIFED Equation Structure**

Household Demands

$$(1) \quad x_{(is)}^{(C)r} = \varepsilon_{(is)}^r c^r + \sum_{k=1} \sum_{t=1} \eta_{(is)(kt)}^r p_{(kt)} \quad \begin{matrix} i,s=1,2,3 \\ r=1,2 \end{matrix}$$

Foreign Demand

$$(2) \quad p_{(ir)}^{(X)} = -\gamma_{(ir)} x_{(ir)}^{(X)} + f_{(ir)}^{(X)} \quad \begin{matrix} i,s=1,2,3 \\ r=1,2 \end{matrix}$$

Federal Government Demands

$$(3) \quad x_{(is)}^{(FG)} = c_R + f^{(FG)} \quad i,s=1,2,3$$

Regional Government Demands

$$(4) \quad x_{(is)}^{(RG)r} = x^{(RG)r} \quad \begin{matrix} i,s=1,2,3 \\ r=1,2 \end{matrix}$$

Regional Industry Demands for Intermediate Inputs

$$(5) \quad x_{(is)}^{jr} = x_j^r - (p_{(is)} - \sum_{t=1}^3 \alpha_{(it)}^{jr} p_{(it)}) \quad i,s=1,2,3$$

Regional Industry Demands for Primary Factors

$$(6) \quad x_{(Fv)}^{jr} = x_j^r - (p_{(Fv)}^{jr} - \sum_q \alpha_{(Fq)}^{jr} p_{(Fq)}^{jr}) \quad \begin{matrix} j=1,2,3 \\ r,v=1,2 \end{matrix}$$

Zero Pure Profits in Production

$$(7) \quad p_{(jr)} = \sum_{i=1}^3 \sum_{s=1}^3 S_{(is)}^{jr} p_{(is)} + \sum_{v=1}^2 S_{(Fv)}^{jr} p_{(Fv)}^{jr} + S_{(CT)}^{jr} p_{(CT)jr} - S_{(ST)}^{jr} p_{(ST)jr} \quad \begin{matrix} j=1,2,3 \\ r=1,2 \end{matrix}$$

Zero Pure Profits in Importing

$$(8) \quad p_{(i3)} = p_{(i3)}^{(F)} + \tau_i t_i + \phi \quad i=1,2,3$$

Zero Pure Profits in Exporting

$$(9) \quad p_{(ir)}^{(X)} + \phi = p_{(ir)} \quad \begin{matrix} i=1,2,3 \\ r=1,2 \end{matrix}$$

Marketing Clearing for Commodities

$$(10) \quad x_i^r = \sum_{j=1}^3 \sum_{t=1}^2 B_{(ir)}^{jt} x_{(ir)}^{jt} + \sum_{t=1}^2 B_{(ir)}^{(C)t} x_{(ir)}^{(C)t} + B_{(ir)}^{(X)} x_{(ir)}^{(X)} + B_{(ir)}^{(FG)} x_{(ir)}^{(FG)} + \sum_{t=1}^2 B_{(ir)}^{(RG)t} x_{(ir)}^{(RG)t} \quad \begin{matrix} i=1,2,3 \\ r=1,2 \end{matrix}$$

Labour market balance

$$(11) \quad \sum_{j=1}^3 W^{jr} x_{(F1)}^{jr} = \ell^r \quad r=1,2$$

Demand equals supply for services of sector-specific capital

$$(12) \quad x_{(F2)}^{jr} = k_j^r \quad \begin{matrix} j=1,2,3 \\ r=1,2 \end{matrix}$$

Foreign-currency value of imports

$$(13) \quad m = \sum_{i=1}^3 S_i^{(M)} \left( p_{(i3)}^{(F)} + \sum_{j=1}^3 \sum_{r=1}^2 S_i^{(M)jr} x_{(i3)}^{jr} + \sum_{r=1}^2 S_i^{(MC)r} x_{(i3)}^{(C)r} + \sum_{r=1}^2 S_i^{(MG)r} x_{(i3)}^{(RG)r} \right)$$

Foreign-currency value of exports by region

$$(14) \quad e^r = \sum_{i=1}^3 S_i^{(X)r} (p_{(ir)}^{(X)} + x_{(ir)}^{(X)}) \quad r=1,2$$

Foreign-currency value of exports in aggregate

$$(15) \quad e = \sum_{r=1}^2 S^{(X)r} e^r$$

National trade balance

$$(16) \quad \Delta B = (Ee - Mm)/100$$

Domestic-currency value of interregional imports

$$(17) \quad m_s^r = \sum_{i=1}^3 \sum_{j=1}^3 S_{(is)}^{(IM1)jr} (p_{(is)} + x_{(is)}^{jr}) + \sum_{i=1}^3 S_{(is)}^{(IM2)r} (p_{(is)} + x_{(is)}^{(C)r}) + \sum_{i=1}^3 S_{(is)}^{(IM3)r} (p_{(is)} + x_{(is)}^{(RG)r}) \quad \begin{matrix} r,s=1,2 \\ r \neq s \end{matrix}$$

Interregional domestic trade balance

$$(18) \quad 100 \Delta B_s^r = M_r^s m_r^s - M_s^r m_s^r \quad \begin{matrix} r,s=1,2 \\ r \neq s \end{matrix}$$

Overseas imports by region

$$(19) \quad m_3^r = \sum_{i=1}^3 S_i^{(FM)r} (p_{(i3)}^{(F)} + \sum_{j=1}^3 S_i^{(FM)jr} x_{(i3)}^{jr}) + S_i^{(FMC)r} x_{(i3)}^{(C)r} + S_i^{(FMG)r} x_{(i3)}^{(RG)r} \quad r=1,2$$

Regional balance of trade (international and interregional)

$$(20) \quad \Delta B^r = \Delta B_{s(\neq r,3)}^r + (\Phi/100)[E^r e^r - M_3^r m_3^r + (E^r - M_3^r)\Phi] \quad r=1,2$$

Consumer price index

$$(21) \quad cpi = \sum_{r=1}^2 S^{(C)r} (cpi)^r$$

Regional consumer price index

$$(22) \quad (cpi)^r = \sum_{i=1}^3 \sum_{s=1}^3 S_{(is)}^{(C)r} p_{(is)}$$

Producer wage cost per labour unit

$$(23) \quad p_{(F1)}^{jr} = p_{(F1)}^r + (SW)^{jr} t_{(PR)}^{jr} \quad \begin{matrix} j=1,2,3 \\ r=1,2 \end{matrix}$$

Wage indexation

$$(24) \quad p_{(F1)}^r = h(cpi) + f_{(F1)}^r \quad r=1,2$$

Regional consumption

$$(25) \quad c^r = \sum_{j=1}^3 (p_{(F1)}^{jr} + x_{(F1)}^{jr})(SC)_1^{jr} + \sum_{j=1}^3 \sum_{t=1}^2 (p_{(F2)}^{jt} + x_{(F2)}^{jt})(SC)_2^{jtr} + v_{(UB)}^r (SC)_3^r - t_{(Y)}^r (SC)_4^r \quad r=1,2$$

Regional unemployment level

$$(26) \quad x_{(U)}^r = -(UN)^r \ell^r + (LF)^r f^r \quad r=1,2$$

Unemployment benefits

$$(27) \quad v_{(UB)}^r = (cpi) + x_{(U)}^r \quad r=1,2$$

Income tax

$$(28) \quad t_{(Y)}^r = p_{(F1)}^{(r)} + f_{(LT)} + \ell^r \quad r=1,2$$

Federal government expenditure

$$(29) \quad (fe) = \sum_i \sum_s S_{(is)}^{(FG1)} (p_{(is)} + x_{(is)}^{(FG)}) + \sum_r S^{(FG2)r} v_{(UB)}^r + \sum_r S^{(FG3)r} t_{(IG)}^r \quad r=1,2$$

Inter-governmental grants

$$(30) \quad t_{(IG)}^r = (cpi) + f_{(IG)}^r \quad r=1,2$$

## Federal government revenue

$$(fr) = \sum_{r=1}^2 S^{(FR1)r} t_{(Y)}^r + \sum_{i=1}^3 \sum_{j=1}^3 \sum_{r=1}^2 S_i^{(FR2)jr} (t_i + p_{(i3)}^{(F)} + \phi + x_{(i3)}^{jr})$$

$$(31) \quad + \sum_{i=1}^3 \sum_{r=1}^2 S_i^{(FR3)r} (t_i + p_{(i3)}^{(F)} + x_{(i3)}^{(C)r} + \phi) + \sum_{j=1}^3 \sum_{r=1}^2 S^{(FR4)jr} (p^{(CT)jr} + x_j^r)$$

## Federal government borrowing requirement

$$(32) \quad \Delta B_{(FG)} = (FG)(fe) - (FR)(fr)$$

## Regional government expenditure

$$(ge)^r = \sum_i \sum_s S_{(is)}^{(RG1)r} (p_{(is)} + x_{(is)}^{(RG)r})$$

$$(33) \quad + \sum_i S^{(RG2)jr} (p^{(ST)jr} + x_j^r) \quad r=1,2$$

## Regional government revenue

$$(34) \quad (gr)^r = S_{jr}^{(RR1)r} (t_{(PR)}^{jr} + p_{(F1)}^r + x_{(F1)}^{jr}) + S^{(RR2)r} (cpi + f_{(IG)}^r) \quad r=1,2$$

## Regional governments' borrowing requirements

$$(35) \quad 100 \Delta B_{(RG)} = (GE)^r (ge)^r - (GR)^r (gr)^r \quad r=1,2$$

## APPENDIX B: MINIFED Percentage Change Variables

Variable	Subscript	Number	Description
$x_{(is)}^{(C)r}$	$i, s = 1, 2, 3$ $r = 1, 2$	18	Region $r$ household demand for commodity $i$ from source $s$ .
$c^r$	$r = 1, 2$	2	Aggregate nominal household expenditure in region $r$ .
$p_{(kt)}$	$k, t = 1, 2, 3$	9	Price of commodity $k$ from source $t$ .
$p_{(ir)}^{(X)}$	$i = 1, 2, 3$ $r = 1, 2$	6	F.o.b. foreign currency export price for good $i$ originating from region $r$ .
$x_{(ir)}^{(X)}$	$i = 1, 2, 3$ $r = 1, 2$	6	Export volume of $i$ produced in region $r$ .
$f_{(ir)}^{(X)}$	$i = 1, 2, 3$ $r = 1, 2$	6	Shift variable for foreign export demands.
$x_{(is)}^{(FG)}$	$i, s = 1, 2, 3$	9	Federal government demand for good $i$ from source $s$ .



$c_R$		1	Economy-wide real aggregate household expenditure.
$f^{(FG)}$		1	Shift term for federal government expenditure.
$x_{(is)}^{(RG)r}$	$i, s = 1, 2, 3$ $r = 1, 2$	18	Region $r$ government demand for good $i$ from source $s$ .
$x^{(RG)r}$	$r = 1, 2$	2	Aggregate region $r$ government expenditure.
$x_{(is)}^{jr}$	$i, j, s = 1, 2, 3$ $r = 1, 2$	54	Input demands for current production.
$x_j^r$	$j = 1, 2, 3$ $r = 1, 2$	6	Region $r$ output of good $j$ .
$x_{(Fv)}^{jr}$	$j = 1, 2, 3$ $r, v = 1, 2$	12	Regional industry $jr$ 's demand for primary factor $v$ .
$p_{(Fv)}^{jr}$	$j = 1, 2, 3$ $r, v = 1, 2$	12	Price paid by regional industry $jr$ for primary factor $v$ .
$p^{(CT)jr}$	$j = 1, 2, 3$ $r = 1, 2$	6	Federal government production taxes.
$p^{(ST)jr}$	$j = 1, 2, 3$ $r = 1, 2$	6	Regional government industry subsidies
$p_{(i3)}^{(F)}$	$i = 1, 2, 3$	3	C.i.f. foreign currency import prices
$t_i$	$i = 1, 2, 3$	3	Ad valorem import duties.
$\phi$		1	Exchange rate, domestic currency cost of foreign unit of currency.
$l^r$	$r = 1, 2$	2	Regional employment
$k_j^r$	$j = 1, 2, 3$ $r = 1, 2$	6	Regional industry capital stock.
$m$		1	Foreign currency value of imports.
$e^r$	$r = 1, 2$	2	Foreign currency value of exports from region $r$ .
$e$		1	Foreign currency value of exports.
$\Delta B$		1	National trade balance.
$m_s^r$	$r, s = 1, 2$ $r \neq s$	2	Domestic currency value of interregional imports.
$\Delta B_s^r$	$r, s = 1, 2$ $r \neq s$	2	Interregional domestic trade balance.

$m_3^r$	$r = 1, 2$	2	Aggregate overseas imports into region $r$ .
$\Delta B^r$	$r = 1, 2$	2	Local currency balance of all trade for region $r$ .
$cpi$			Economy-wide consumer price index.
$(cpi)^r$	$r = 1, 2$	2	Region $r$ consumer price index.
$p_{(F1)}^r$	$r = 1, 2$	2	Pre-tax consumer wage.
$t_{(PR)}^{jr}$	$j = 1, 2, 3$ $r = 1, 2$	6	Payroll tax per labour unit.
$f_{(F1)}^r$	$r = 1, 2$	2	Regional wage shift variable.
$v_{(UB)}^r$	$r = 1, 2$	2	Value of unemployment benefits paid to region $r$ residents.
$t_{(Y)}^r$	$r = 1, 2$	2	Total income tax paid by region $r$ residents.
$x_{(U)}^r$	$r = 1, 2$	2	Regional unemployment level.
$f$	$r = 1, 2$	2	Region $r$ labour force.
$f_{(LT)}$		1	Income tax per labour unit.
$(fe)$		1	Federal government expenditure.
$t_{(IG)}^r$	$r = 1, 2$	2	Inter-governmental grants.
$f_{(IG)}^r$	$r = 1, 2$	2	Shift variable for inter-governmental grants.
$(fr)$		1	Federal government revenue.
$\Delta B_{(FG)}$		1	Federal government borrowing requirement.
$(ge)^r$	$r = 1, 2$	2	Region $r$ government expenditure.
$(gr)^r$	$r = 1, 2$	2	Region $r$ government receipts.
$\Delta B_{(RG)}^r$	$r = 1, 2$	2	Region $r$ government's borrowing requirement.

#### APPENDIX C. List of MINIFED Coefficients and Parameters

Equation	Coefficient or Parameter	Description
(1)	$\varepsilon^r$	Region $r$ household expenditure elasticity.

	$\eta_{(is)(kt)}^r$	Cross price elasticity of region $r$ household demand for good $i$ from source $s$ with respect to changes in the price of good $k$ from source $t$ .
(2)	$\gamma_{(ir)}$	Reciprocal of the elasticity of demand for region $r$ good $i$ with respect to its foreign currency price.
(3)	none	
(4)	none	
(5)	$\alpha_{(it)}^{jr}$	Share of source $t$ in all sources of good $i$ being purchased by industry $j$ located in region $r$ .
(6)	$\alpha_{(Fq)}^{jr}$	Share of primary factor $q$ in regional industry $jr$ 's purchase of all primary factors.
(7)	$S_{is}^{jr}$	Share of good $i$ from source $s$ in total cost of production of $jr$ .
	$S_{Fv}^{jr}$	Share of cost of primary factor $v$ in total production costs.
	$S_{(CT)}^{jr}$	Share of production tax costs in total production costs.
	$S_{(ST)}^{jr}$	Share of subsidy value in total production costs.
(8)	$\tau_i$	Ratio of duty to duty-laden price.
(9)	none	
(10)	$B_{ir}^{jt}$	Share of sales of good $i$ from source $r$ absorbed by intermediate demands of industry $j$ in region $r$ .
	$B_{(ir)}^{(CT)}$	Sales share of $ir$ absorbed by households in region $t$ .
	$B_{(ir)}^{(X)}$	Sales share of $ir$ absorbed by exports overseas.
	$B_{(ir)}^{(FG)}$	Sales share of $ir$ to federal government.
	$B_{(ir)}^{(RG)t}$	Sales share of $ir$ to regional government $t$ .
(11)	$W^{jr}$	Industry $j$ share in total regional employment.
(12)	none	
(13)	$S_i^{(M)}$ $S_i^{(MC)r}$	Share of import $i$ accounted for by region $r$ households.

- (14)  $S_i^{(MG)}$  Share of import  $i$  accounted for by federal government.
- $S_i^{(MG)r}$  Share of import  $i$  accounted for by region  $r$  government.
- (15)  $S_i^{(X)r}$  Share of commodity  $i$  produced in region  $r$  in total international exports from the region.
- (16)  $S^{(X)r}$  Region  $r$  share in total exports abroad.
- (17)  $E$  Base-period value of exports.
- $M$  Base-period value of imports.
- (18)  $S_{is}^{(IM1)jr}$  Share of total interregional imports by region  $r$  of commodity  $i$  from region  $s$  accounted for by regional industry  $jr$ .
- $S_{is}^{(IM2)jr}$  Share of region  $r$  interregional imports of  $i$  accounted for by households.
- $S_{is}^{(IM3)jr}$  Share of region  $r$  interregional imports of  $i$  accounted for by the regional government.
- (19)  $M_r^s$  Base-year value of aggregate interregional imports by region  $s$  from region  $r$ .
- $S_i^{(FM)r}$  Share of commodity  $i$  imports in total foreign imports by region  $r$ .
- $S_i^{(FM)jr}$  Share of region  $r$  foreign imports of  $i$  accounted for by  $jr$  purchases.
- $S_i^{(FMC)r}$  Share of foreign imports of  $i$  into region  $r$  accounted for by household purchases.
- $S_i^{(FMG)r}$  Share of foreign imports of  $i$  into region  $r$  accounted for by regional government purchases.
- (20)  $\Phi$  Base-year exchange rate, domestic currency units per foreign currency units.
- $E^r$  Base-period value of region  $r$  aggregate exports overseas.
- $M_e^r$  Base-period value of region  $r$  foreign imports.
- (21)  $S^{(C)r}$  Share of value of national household consumption accounted for by household consumption in region  $r$ .

(22)	$S_{(is)}^{(C)r}$	Share of region $r$ representative household's total budget spent on commodity $i$ from source $s$ .
(23)	$(SW)^r$	Share of producer wage per labour unit employed in regional industry $jr$ accounted for by payroll tax.
(24)	$h$	Level of wage-indexation.
(25)	$(SC)_1^{jr}$	Share of labour income earned in regional industry $jr$ in region $r$ disposable income.
	$(SC)_2^{jtr}$	Share of returns to capital, owned by region $r$ residents and located in regional industry $jt$ , in total region $r$ disposable income
	$(SC)_3^r$	Share of unemployment benefits in total region $r$ disposable income.
	$(SC)_4^r$	Ratio of income taxes levied on region $r$ residents to total region $r$ disposable income.
(26)	$(UN)^r$	Ratio of employment to unemployment in region $r$ .
	$(LF)^r$	Ratio of labour force to unemployment in region $r$ .
(27)	none	
(28)	none	
(29)	$S_{is}^{(FG1)}$	Share in aggregate federal government expenditure of current expenditure on good $i$ from source $s$ .
	$S^{(FG2)r}$	Share in aggregate federal government expenditure of unemployment benefits.
	$S^{(FG3)r}$	Share in aggregate federal government expenditure of grants to region $r$ government.
(30)	none	
(31)	$S^{(FR1)r}$	Share in total federal government revenue of income taxes.
	$S_i^{(FR2)jr}$	Share in total federal government revenue of tariff revenue on imports of good $i$ by regional industry $jr$ .
	$S_i^{(FR3)jr}$	Share in total federal government revenue of tariff revenue on imports of good $i$ by region $r$ households.



	$S^{(FR4)jr}$	Share in total federal government revenue of production taxes levied on regional industry $jr$ .
(32)	$(FG)$	Base-period value of total federal government outlays.
	$(FR)$	Base-period value of total federal government revenue.
(33)	$S_{(is)}^{(RG1)r}$	Share in total region $r$ government expenditure of current expenditure on good $i$ from source $s$ .
	$S^{(RG2)jr}$	Share in total region $r$ government expenditure of subsidies to regional industry $jr$ .
(34)	$S_j^{(RR1)r}$	Share in total region $r$ government revenue of payroll taxes on industry $j$ .
	$S^{(RR2)r}$	Share in total region $r$ government revenue of grants from the federal government.
(35)	$(GE)^r$	Base-period region $r$ government aggregate expenditure.
	$(GR)^r$	Base-period region $r$ government aggregate revenue.