# THE INTER AND INTRAREGIONAL EFFECTS OF ALTERNATIVE INFRASTRUCTURE LOCATION DECISIONS

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**ABSTRACT** The location of new infrastructure is often a matter of interregional competition. In a multiregional context where there exist linkages between regions a comprehensive resolution of the specific regional location of a new facility may not clearly favour the most directly obvious region. In this paper, recent results on inverse matrix decomposition and structural path analysis are employed to examine this issue. The analysis demonstrates that the ability of an infrastructural development to strengthen a region's internal intersectoral multiplier is not necessarily related to the proximity of the infrastructure but depends crucially on links between the infrastructure and the region in which it is located as well as on regional trading linkages. An example which looks at the effects of alternative locations of new infrastructure is used, firstly, to demonstrate that the ultimate effects of locational development are complex and may produce surprising results and, secondly, to highlight the types of relationships which, in a multiregional input-output modelling context, hold the key to provision of a quantitative solution to the effects-of-location question.

## 1. INTRODUCTION

The location of new infrastructure is often a matter of interregional competition. Entering that competition may be seen as a risky investment. Yet in a multiregional context with sufficient linkages between regions the question can be asked, how can one determine to what extent the specific regional location of a new facility matters? Given linkages between regions, and given that some regions will at any point in time have greater intraregional strength than others, it is not immediately apparent what the ultimate benefits are for a region of having a facility developed in its own vicinity, for example, versus enjoying the spillover effects of a facility located in another region which may be better placed, by its existing structure, to generate extensive multiplier effects.

The opportunity to obtain advantage from possibly enhanced multiplier effects in another region will, of course, depend upon the nature of linkages. These may be quite indirect and not immediately obvious either in nature or quantitative impact. When these issues are considered more carefully, it becomes clear that, while location may matter, it is not a foregone conclusion that proximity matters.

One way to respond to the issues raised above is to examine the data. It is clear that, from the nature of the problem, a multiregional input-output model ought

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logically to be part of the analytical apparatus which should be brought to bear on this. These models are, however, expensive to produce. In all likelihood some pre-existing model, developed for another purpose, will need to be utilised. It may well be the case that multiregional linkages in such a model, if they exist at all, are determined by a mix of assumptions and relevant data inspection. In this case, in order to have some confidence in the predictions of the model, it would be useful to know what are the crucial features of the model which generate its specific conclusions.

Examination of the extent to which relevant data versus assumptions went into the construction of the identified crucial components of the model would be especially helpful in determining whether the analysis could be relied upon in the context of the use of the existing model or whether more basic model reconstruction should be undertaken prior to further analysis.

In this paper, recent results on inverse matrix decomposition and structural path analysis are employed to illustrate their applicability in providing relevant model assessment information in the context described above. An example which looks at the effects of alternative locations of new infrastructure is used, firstly, to demonstrate that the ultimate effects of locational development are complex and may produce surprising results and, secondly, to highlight the types of relationships which, in a multiregional input-output modelling context, hold the key to provision of a quantitative solution to the effects-of-location question.

Matrix decomposition and structural path techniques have been used to investigate a variety of issues of interest in which a multiregional or multi-account input-output model is a vehicle for analysis. Yet, surprisingly, the developmental aspects of infrastructure location decisions do not seem to have been examined using these techniques. This paper employs the conjunction of a specific decomposition with an analysis of alternative paths of transmission of economic influence which are revealed by the decomposition. This allows some of the implications of alternative facility location proposals to be clearly analysed.

In the next section of the paper, relevant results from inverse matrix decomposition and structural path analysis are summarised. In Section 3, these results are re-characterised and interpreted in an appropriate context for the currently proposed type of analysis. Section 4 provides an illustration and Section 5 draws out some policy implications.

## 2. PRELIMINARY RESULTS

In this section, two recently developed results on multiplier decompositions are presented in a unified format as a basis for the key analytical result to be exploited in the current paper. The two relevant results are Proposition 3 from Cooper (1998) and Proposition 2 from Cooper (1999). To summarise these in a format suitable for application in the current context, let:

$$A[n] \equiv \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix} \tag{1}$$

denote an n x n partition of an input-output matrix, and let:

$$A[n]^{(r)} \equiv \begin{bmatrix} A_{11}^{(r)} & \cdots & A_{1n}^{(r)} \\ \vdots & & \vdots \\ A_{n1}^{(r)} & \cdots & A_{nn}^{(r)} \end{bmatrix} \tag{2}$$

denote a commensurately partitioned matrix constructed from the underlying matrix $A[n]$ as follows. Each block of $A[n]^{(r)}$ is recursively defined, for $r, j, k = 1,\ldots,n$, by relationships of the form:

$$A_{jk}^{(r)} = A_{jk}^{(r-1)} + A_{jr}^{(r-1)}\Delta_r^{(r-1)}A_{rk}^{(r-1)}, \quad \Delta_r^{(r-1)} = \left\{I - A_{rr}^{(r-1)}\right\}^{-1}, \quad A_{jk}^{(0)} = A_{jk}. \tag{3}$$

<u>Proposition 1</u>: *Recursively Based Additive-Multiplicative Decomposition of the Leontief Inverse*

Let $A[n]$, $A[n]^{(r)}$ be defined by (1) and (2) respectively. Let $A[n]_{j\bullet}^{(r)}$ and $A[n]_{\bullet k}^{(r)}$ denote the j$^{\text{th}}$ row block and k$^{\text{th}}$ column block of $A[n]^{(r)}$ respectively. Let:

$$B[n] \equiv \{I - A[n]\}^{-1} \tag{4}$$

denote the Leontief inverse of $A[n]$. Then:

$$B[n] = I + A[n]^{(n)} \tag{5}$$

where $$A[n]^{(r)} = A[n]^{(r-1)} + \Delta A[n]^{(r)} \qquad \text{for r = 1, …, n} \tag{6}$$

and $$\Delta A[n]^{(r)} = A[n]_{\bullet r}^{(r-1)} \; \Delta_r^{(r-1)} \; A[n]_{r\bullet}^{(r-1)}. \tag{7}$$

*Proof:* Cooper (1998, Proposition 3) shows that $B[n]_{jk} = \delta_{jk}I + A_{jk}^{(n)}$ where $A_{jk}^{(n)}$ is defined by (3) for r = n and $\delta_{jk}$ is the Kronecker delta. Given definitions (2), (6) and (7), result (5) is simply a matrix expression of this result. #

Before making use of this result in the current context, it will be useful to interpret it and to compare it with other decomposition approaches in the literature.

In Cooper (1999, Proposition 2), results (5) and (6) are expressed in the combined form $B[n] = I + A[n] + \sum_{r=1}^{n} \Delta A[n]^{(r)}$. Given (7) above, the formulation employed in Cooper (1999) lays emphasis on the fact that the decomposition has a nested additive / multiplicative structure. Each additive term may be interpreted as the increment to the inverse due to consideration of additional row and column blocks at the margin.

As (7) demonstrates, each additive term has a tripartite multiplicative decomposition. In Cooper (1999) this was written as $\Delta A[n]^{(r)} = \hat{A}[n]_{\bullet r}^{(r-1)}\left[i_n i_n' \otimes \Delta_r^{(r-1)}\right]\hat{A}[n]_{r\bullet}^{(r-1)}$, where ^ denotes a block diagonal matrix formed from the row or column block operated upon, $i_n$ is a n-dimensional unit vector and $\otimes$ is the Kronecker product. This form represents $\Delta A[n]^{(r)}$ as a product of a block diagonal matrix, a full (interregional) multiplier dispersion matrix and a second block diagonal matrix.

The purpose of the specification in Cooper (1999) was to afford a comparison with a reasonably extensive literature in which it is shown that tripartite multiplicative decompositions of the Leontief inverse exist with the multiplier matrices having this type of block diagonal / off diagonal dispersion / block diagonal structure. Pyatt and Round (1979), Defourny and Thorbecke (1984), Crama, Defourny and Gazon (1984), Round (1985, 1988, 1989), Sonis and Hewings (1988, 1990, 1993, 1998a, 1998b), Sonis, Hewings and Gazel (1995), Sonis, Hewings and Lee (1994) and Sonis, Hewings and Miyazawa (1997) all discuss these types of decompositions and exhibit special cases of them.

However the quoted literature also shows that, except for low dimensional cases, the structure of the individual blocks within the matrices making up the multiplicative decomposition is typically quite complex and does not readily lead to straightforward interpretations in terms of paths of influence. In higher dimensional cases, for example, where most of the results are due to Sonis and Hewings (1988, 1990, 1993, 1998a, 1998b), the alternative proposed decompositions all involve particular sub-matrices appearing a number of times (depending upon the decomposition technique) in perhaps several of the component matrices in the multiplicative decomposition. Furthermore, for many of these proposed decompositions, the order of the matrix products is not unique. These properties make it difficult to interpret the sub-matrices as nodes in a structural path exhibited by a natural direction of influence through an easily interpreted path implied by the product of the matrices in the multiplicative decomposition.

By nesting multiplicative relationships within a natural additive form, Cooper (1999) shows that this specification allows for a simpler interpretation in terms of paths of influence. The current paper exploits this simplicity to construct and interpret the paths of influence implied by the change in the Leontief inverse as a new relationship is entered into. In view of the particular focus of the current paper, which is to construct and examine specific paths of influence, it is convenient to exploit the special structure of the multiregional multiplier dispersion matrix

$i_n i_n' \otimes \Delta_3^{(2)}$ in the term $\Delta A[n]^{(r)}$, which was derived in Cooper (1999), to collapse this formulation to the current specification (7).

In the next section, special features of the additive / multiplicative decomposition are exploited to recast the decomposition into one representing the change in the Leontief multiplier as the effect of location of a new piece of infrastructure is taken into account.

## 3. A GENERAL FORMULATION FOR MULTIREGIONAL MULTIPLIER EFFECTS OF INFRASTRUCTURAL DEVELOPMENT

A convenient way to analyse the alternative implications of new infrastructure provision in one of several possible regions is to construct a multiregional model and then extend it to include a new, artificial, "region" which represents the new infrastructure. The provision of linking relationships between the artificial region and any one of the original regions then serves to represent actual location in that region. Linkages with other regions take place indirectly, via those other regions' existing links with the favoured region.

In setting up an extension of the model via the provision of another "region" to represent the new infrastructure, the question arises as to the complexity of this "region". If the infrastructure is relatively simple, it might be represented by one new row and column in the multiregional input-output matrix. However, to allow for more complex infrastructure involving a variety of activities (an airport and its associated business activities, for example), it is useful to construct the new region as a complete new block with multiple rows and columns, not necessarily commensurate with but in principle similar to the row and column blocks assigned to real individual regions. In a complex piece of infrastructure, one might also wish to allow for intra "regional" activity, while in a less complex case the direct intra "regional" block may be null.

The importance of the recent developments in matrix decomposition analysis which have been summarised in the previous section is that they allow the influence of the structure of the block partitioned input-output coefficient matrix to be traced through to the structure of the Leontief inverse. However, it is useful to develop that analytical machinery a little further to compare the extended model with the initial (base case or no infrastructure) model so that the effect of the existence of the new infrastructure, wherever it is located, can be assessed.

In the current context, the Leontief inverse needs to be first analysed for the base case model and then re-analysed for the model extension in which the new block has direct links with one particular region and with provision for internal direct intra "regional" links in the new block. The strategy in this section is to develop a general formulation which can be specialised to examine particular linkages.

To allow for these considerations, let (1) for n = m denote the initial multiregional situation and let (1) for n = m+1 denote the extended model which sets up the new infrastructure as "region" m+1. The partitioned Leontief inverses for the initial and extended models are defined by (4) for n = m and n = m+1 respectively. Letting $\Delta B[m+1]$ denote the matrix of changes in the Leontief inverse as a result of the model extension, this matrix may be defined as:

$$\Delta B[m+1] \equiv B[m+1] - \left[\begin{array}{c|c} B[m] & 0 \\ \hline 0 & 0 \end{array}\right]. \tag{8}$$

A useful analytical device would be a formula for directly assessing the changes in the Leontief inverse which occur as a result of a model extension of the type envisaged here. To construct this, it is helpful to define the concept of a truncated row and column block matrix as follows. Let

$$M[n] \equiv \begin{bmatrix} M_{11} & \cdots & M_{1n} \\ \vdots & & \vdots \\ M_{n1} & \cdots & M_{nn} \end{bmatrix}$$

denote an arbitrary matrix with an n x n partitioned structure. Let $M[n]_{j\bullet}$ and $M[n]_{\bullet k}$ denote the j[th] row block and the k[th] column block of $M[n]$ respectively, consistent with the notation employed in Proposition 1. Now define

$$M[n]_{j,*} \equiv M[n]_{j\bullet} \begin{bmatrix} I[n-1] \\ 0 \end{bmatrix} \text{ and} \tag{9}$$

$$M[n]_{*,k} \equiv \begin{bmatrix} I[n-1] & 0 \end{bmatrix} M[n]_{\bullet k} \tag{10}$$

where $I[n-1]$ denotes an (n-1)-block identity matrix conformable with the first n-1 blocks in $M[n]$. By their definition, these structures may be written as:

$$M[n]_{j,*} = \begin{bmatrix} M_{j1} & \cdots & M_{j,n-1} & M_{jn} \end{bmatrix} \begin{bmatrix} I & & \\ & \ddots & \\ & & I \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} M_{j1} & \cdots & M_{j,n-1} \end{bmatrix} \text{ and} \tag{9'}$$

$$M[n]_{*,k} = \begin{bmatrix} I & & & 0 \\ & \ddots & & \vdots \\ & & I & 0 \end{bmatrix} \begin{bmatrix} M_{1k} \\ \vdots \\ M_{n-1,k} \\ M_{nk} \end{bmatrix} = \begin{bmatrix} M_{1k} \\ \vdots \\ M_{n-1,k} \end{bmatrix} \tag{10'}$$

The elaborations (9') of (9) and (10') of (10) show that these matrix constructions are, respectively, the truncated j[th] row block and truncated k[th] column block of $M[n]$. Using these constructions, the following proposition, which is the key analytical result of the current paper, sets up the relationships necessary to analyse the effect of a model extension of the type discussed above.

Proposition 2: *Multiplicative Decomposition of the Extended Model Change in the Leontief Inverse*

Let $A[m]$ and $A[m]^{(r)}$ be defined by (1) and (2) respectively for n = m partitions.

Let $A[m+1]$ and $A[m+1]^{(r)}$ denote the equivalent constructs for an extended model in which there are m+1 partitions.

Let $B[m]$ and $B[m+1]$ denote the respective Leontief inverses, defined by (4), for the two cases n = m and n = m+1.

Let $A_{m+1,*}^{(m)}$ and $A_{*,m+1}^{(m)}$ respectively denote the truncated (m+1)[th] row block and the truncated (m+1)[th] column block of $A[m+1]^{(m)}$.

Let $\Delta B[m+1]$ be defined by (8).

Then the changes in the Leontief inverse have the tripartite multiplicative representation:

$$\Delta B[m+1] = \begin{bmatrix} A_{*,m+1}^{(m)} \\ I \end{bmatrix} \Delta_{m+1}^{(m)} \begin{bmatrix} A_{m+1,*}^{(m)} & I \end{bmatrix}. \tag{11}$$

*Proof*: The proof uses the fact that, by definition, $\Delta_{m+1}^{(m)} = \left\{ I - A_{m+1,m+1}^{(m)} \right\}^{-1}$, which implies that $\Delta_{m+1}^{(m)} = I + \Delta_{m+1}^{(m)} A_{m+1,m+1}^{(m)} = I + A_{m+1,m+1}^{(m)} \Delta_{m+1}^{(m)}$ (see, for example, Henderson and Searle, 1981, eqn. 20). For present purposes, this has three simplifying implications, all of which exploit definition (3):

(i) $$A_{*,m+1}^{(m+1)} \equiv A_{*,m+1}^{(m)} + A_{*,m+1}^{(m)} \Delta_{m+1}^{(m)} A_{m+1,m+1}^{(m)} = A_{*,m+1}^{(m)} \Delta_{m+1}^{(m)}$$

(ii) $$A_{m+1,*}^{(m+1)} \equiv A_{m+1,*}^{(m)} + A_{m+1,m+1}^{(m)} \Delta_{m+1}^{(m)} A_{m+1,*}^{(m)} = \Delta_{m+1}^{(m)} A_{m+1,*}^{(m)}$$

(iii) $$A_{m+1,m+1}^{(m+1)} \equiv A_{m+1,m+1}^{(m)} + A_{m+1,m+1}^{(m)} \Delta_{m+1}^{(m)} A_{m+1,m+1}^{(m)} = \Delta_{m+1}^{(m)} A_{m+1,m+1}^{(m)} = -I + \Delta_{m+1}^{(m)}$$

The proof also exploits the modular definition of (2), which implies that, for n = m+1:

$$A[m+1]^{(r)} = \left[\begin{array}{c|c} A[m]^{(r)} & A^{(r)}_{*,m+1} \\ \hline A^{(r)}_{m+1,*} & A^{(r)}_{m+1,m+1} \end{array}\right].$$

Using these intermediate results and the basic definition (8), one obtains:

$$\begin{aligned}
\Delta B[m+1] &\equiv B[m+1] - \left[\begin{array}{c|c} B[m] & 0 \\ \hline 0 & 0 \end{array}\right] \\
&= I + A[m+1]^{(m+1)} - \left[\begin{array}{c|c} I + A[m]^{(m)} & 0 \\ \hline 0 & 0 \end{array}\right] \\
&= \left[\begin{array}{c|c} I + A[m]^{(m+1)} & A^{(m+1)}_{*,m+1} \\ \hline A^{(m+1)}_{m+1,*} & I + A^{(m+1)}_{m+1,m+1} \end{array}\right] - \left[\begin{array}{c|c} I + A[m]^{(m)} & 0 \\ \hline 0 & 0 \end{array}\right] \\
&= \left[\begin{array}{c|c} \Delta A[m]^{(m+1)} & A^{(m+1)}_{*,m+1} \\ \hline A^{(m+1)}_{m+1,*} & I + A^{(m+1)}_{m+1,m+1} \end{array}\right] \\
&= \left[\begin{array}{c|c} A^{(m)}_{*,m+1} \Delta^{(m)}_{m+1} A^{(m)}_{m+1,*} & A^{(m)}_{*,m+1} \Delta^{(m)}_{m+1} \\ \hline \Delta^{(m)}_{m+1} A^{(m)}_{m+1,*} & \Delta^{(m)}_{m+1} \end{array}\right] \\
&= \begin{bmatrix} A^{(m)}_{*,m+1} \\ I \end{bmatrix} \Delta^{(m)}_{m+1} \begin{bmatrix} A^{(m)}_{m+1,*} & I \end{bmatrix}
\end{aligned}$$

which establishes equation (11).

Proposition 2 establishes a tripartite multiplicative decomposition of the changes in the Leontief inverse resulting from extension of an m-regional model to encompass a new artificial "region". This result is exploited in the next section to analyse the effect of an infrastructure location decision in the two regional case.

## 4. ILLUSTRATION: A TWO-REGION COMPETITIVE CASE

The initial two-regional model may be represented by:

$$A[2] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}. \tag{12}$$

Location of new infrastructure in 1 may be represented by the artificially extended model:

$$A[3]\big|_1 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} \tag{13}$$

where the notation $\big|_1$ indicates that the facility is located in Region 1 and therefore that direct linkages $A_{23}$ and $A_{32}$ do not exist. However, a complex pattern of interrelationships within the facility itself is allowed via $A_{33}$. The linkages matrices $A_{13}$ and $A_{31}$ represent the direct relationship between the facility and Region 1 due to the facility's location in Region 1. Indirect relationships between the facility and Region 2 will, of course, be generated through the existing interregional trading relationships between Regions 1 and 2.

It will be useful to define, in an analogous way, notation for the location of the facility in Region 2 as:

$$A[3]\big|_2 = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix}. \tag{14}$$

However, it will be helpful initially to proceed in the general case:

$$A[3] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{15}$$

which will allow later comparative analysis of the two options $A[3]\big|_1$ and $A[3]\big|_2$.

Proposition 2 indicates that the matrix of changes in the Leontief inverse may be constructed from the truncated third row and column blocks of $A[3]^{(2)}$. In turn, Proposition 1 provides a recursive construction of these components through the use of (6) and (7). Starting from $A[3]$ we may construct $\Delta A[3]^{(1)}$ via (7) by utilising the first row and column blocks of $A[3]$. Thus:

$$\Delta A[3]^{(1)} = A[3]_{\bullet 1}\ \Delta_1\ A[3]_{1\bullet} = \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} \Delta_1 \begin{bmatrix} A_{11} & A_{12} & A_{13} \end{bmatrix} \tag{16}$$

where:

$$\Delta_1 = \{I - A_{11}\}^{-1}. \tag{17}$$

Then, by recursive use of results (6) and (7) from Proposition 1 and simplification of expressions using the basic result that, for any matrix $M$, the Leontief inverse of $M$ has alternative representations as $(I-M)^{-1}$ $=I+M(I-M)^{-1}$ $=I+(I-M)^{-1}M$ (see, for example, Henderson and Searle, 1981, eqn. 20), we may successively construct:

$$A[3]^{(1)} = A[3] + \Delta A[3]^{(1)} = \begin{bmatrix} -I+\Delta_1 & \Delta_1 A_{12} & \Delta_1 A_{13} \\ A_{21}\Delta_1 & A_{22}+A_{21}\Delta_1 A_{12} & A_{23}+A_{21}\Delta_1 A_{13} \\ A_{31}\Delta_1 & A_{32}+A_{31}\Delta_1 A_{12} & A_{33}+A_{31}\Delta_1 A_{13} \end{bmatrix} \quad (18)$$

$$\Delta A[3]^{(2)} = A[3]^{(1)}_{\bullet 2}\, \Delta_2^{(1)}\, A[3]^{(1)}_{2\bullet}$$
$$= \begin{pmatrix} \Delta_1 A_{12} \\ A_{22}+A_{21}\Delta_1 A_{12} \\ A_{32}+A_{31}\Delta_1 A_{12} \end{pmatrix} \Delta_2^{(1)} \begin{pmatrix} A_{21}\Delta_1 & A_{22}+A_{21}\Delta_1 A_{12} & A_{23}+A_{21}\Delta_1 A_{13} \end{pmatrix} \quad (19)$$

where:
$$\Delta_2^{(1)} \equiv \left\{I - A_{22}^{(1)}\right\}^{-1} = \left\{I - \left[A_{22}+A_{21}\Delta_1 A_{12}\right]\right\}^{-1}, \quad (20)$$

$$A[3]^{(2)} = A[3]^{(1)} + \Delta A[3]^{(2)}$$
$$= \begin{bmatrix} * & * & \Delta_1 A_{13} + \Delta_1 A_{12}\Delta_2^{(1)}\left(A_{23}+A_{21}\Delta_1 A_{13}\right) \\ * & * & \Delta_2^{(1)}\left(A_{23}+A_{21}\Delta_1 A_{13}\right) \\ A_{31}\Delta_1 + \left(A_{32}+A_{31}\Delta_1 A_{12}\right)\Delta_2^{(1)} A_{21}\Delta_1 & \left(A_{32}+A_{31}\Delta_1 A_{12}\right)\Delta_2^{(1)} & * \end{bmatrix} \quad (21)$$

where, for notational convenience, only the required truncated third row and column terms are displayed in (21).

It follows from Proposition 2, equation (11) that:

$$\Delta B[3]=\begin{bmatrix}\left(\Delta_1+\Delta_1 A_{12}\Delta_2^{(1)}A_{21}\Delta_1\right)A_{13}+\Delta_1 A_{12}\Delta_2^{(1)}A_{23}\\ \\ \Delta_2^{(1)}A_{21}\Delta_1 A_{13}+\Delta_2^{(1)}A_{23}\\ \\ I\end{bmatrix}\Delta_3^{(2)}$$

$$\times\left[\left\{A_{31}\left(\Delta_1+\Delta_1 A_{12}\Delta_2^{(1)}A_{21}\Delta_1\right)+A_{32}\Delta_2^{(1)}A_{21}\Delta_1\right\}\quad\left\{A_{31}\Delta_1 A_{12}\Delta_2^{(1)}+A_{32}\Delta_2^{(1)}\right\}\quad I\right] \tag{22}$$

where:

$$\begin{aligned}\Delta_3^{(2)}&=\left\{I-A_{33}^{(2)}\right\}^{-1}=\left\{I-\left[A_{33}^{(1)}+A_{32}^{(1)}\Delta_2^{(1)}A_{23}^{(1)}\right]\right\}^{-1}\\ &=\left\{I-\left[A_{33}+A_{31}\Delta_1 A_{13}+\left(A_{32}+A_{31}\Delta_1 A_{12}\right)\Delta_2^{(1)}\left(A_{23}+A_{21}\Delta_1 A_{13}\right)\right]\right\}^{-1}\end{aligned} \tag{23}$$

The tripartite decomposition of $\Delta B[3]$ in (22) aids examination of the structural relationships between blocks in $A[3]$ and blocks in $\Delta B[3]$. It should be noted, also, that the apparent asymmetry in the relationships for Regions 1 and 2 in the upper left 2 x 2 set of blocks is entirely due to the notational conventions employed to develop a recursive construction. If Region 2 had been taken as the first region, an unconditional multiplier $\Delta_2$ had been defined for it as $\Delta_2 \equiv \{I - A_{22}\}^{-1}$ and multipliers for Region 1 had been developed conditional on paths through Region 2, then the apparent asymmetrical features with respect to the treatment of Regions 1 and 2 in (22) could be reversed.

## 5. INTERPRETATION AND POLICY IMPLICATIONS

The general results for $\Delta B[3]$, given in equation (22), supported by the multiplier definitions (17), (20) and (23), may be employed to examine the options under which the infrastructure is located either in Region 1 or in Region 2. The relevant comparisons are:

Option 1: Location in Region 1: $A_{23} = 0,\ A_{32} = 0$.

$$\Delta B[3]\big|_1 =$$

$$\begin{bmatrix} \left(\Delta_1 + \Delta_1 A_{12}\Delta_2^{(1)}A_{21}\Delta_1\right)A_{13} \\ \Delta_2^{(1)}A_{21}\Delta_1 A_{13} \\ I \end{bmatrix} \Delta_3^{(2)}\Big|_1 \left[A_{31}\left(\Delta_1 + \Delta_1 A_{12}\Delta_2^{(1)}A_{21}\Delta_1\right) \quad A_{31}\Delta_1 A_{12}\Delta_2^{(1)} \quad I\right] \tag{22'}$$

where: $$\Delta_3^{(2)}\Big|_1 = \left\{I - \left[A_{33} + A_{31}\Delta_1 A_{13} + \left(A_{31}\Delta_1 A_{12}\right)\Delta_2^{(1)}\left(A_{21}\Delta_1 A_{13}\right)\right]\right\}^{-1}. \tag{23'}$$

Option 2: Location in Region 2: $A_{13} = 0,\ A_{31} = 0$.

$$\Delta B[3]\big|_2 = \begin{bmatrix} \Delta_1 A_{12}\Delta_2^{(1)}A_{23} \\ \Delta_2^{(1)}A_{23} \\ I \end{bmatrix} \Delta_3^{(2)}\Big|_2 \left[A_{32}\Delta_2^{(1)}A_{21}\Delta_1 \quad A_{32}\Delta_2^{(1)} \quad I\right] \tag{22''}$$

where: $$\Delta_3^{(2)}\Big|_2 = \left\{I - \left[A_{33} + A_{32}\Delta_2^{(1)}A_{23}\right]\right\}^{-1}. \tag{23''}$$

The interpretation and policy conclusions to follow centre on the strength of a region's own (intraregional) multiplier matrix. Without loss of generality, the analysis is presented from the point of view of Region 2. In particular, the sub-matrix $\Delta B[3]_{22}$ is examined for Options 1 and 2 above. From (22'):

$$\Delta B[3]_{22}\Big|_1 = \Delta_2^{(1)}A_{21}\Delta_1 A_{13}\ \Delta_3^{(2)}\Big|_1\ A_{31}\Delta_1 A_{12}\Delta_2^{(1)} \tag{24'}$$

while from (22"):

$$\Delta B[3]_{22}\Big|_2 = \Delta_2^{(1)}A_{23}\ \Delta_3^{(2)}\Big|_2 A_{32}\Delta_2^{(1)} \tag{24''}$$

A comparison of (24') and (24") suggests that the optimal regional location of the infrastructure depends upon the relative strength of direct versus indirect linkages. If the facility is located in Region 2 then (24") is operable. In this case the gain in the strength of Region 2's internal multiplier depends crucially upon $A_{23}$ and $A_{32}$. These represent, on the one hand, direct calls upon the output of Region 2 by the facility and, on the other hand, usage of the facility by businesses in Region 2. The conditional indirect multiplier $\Delta_3^{(2)}\big|_2$ is also relevant. This is given by (23"). The direct interaction coefficient sub-matrices $A_{23}$ and $A_{32}$ are also crucial for this multiplier.

If the facility is located in Region 1, however, then (24') is operable and Region 2 must rely upon the indirect linkages $A_{21}\Delta_1 A_{13}$ in place of $A_{23}$ and $A_{31}\Delta_1 A_{12}$ in place of $A_{32}$. It is important to note, though, that there is no a priori reason why the indirect linkages would be weaker than the direct links. Indeed, if Region 1 is strong economically, with a sufficiently large internal multiplier matrix $\Delta_1$, then provided there is sufficient trade between Regions 1 and 2 – represented by the size of the coefficient sub-matrices $A_{12}$ and $A_{21}$ – it may well be the case that the indirect effects dominate, and Region 2 would be better served by location of the facility in Region 1. This possibility needs to be considered cautiously, however, because it does not take into account the fact that the indirect conditional multiplier $\Delta_3^{(2)}$ is also dependent on the facility's location. Nevertheless, there is a clear case where the above conclusion can be reached unambiguously.

In the following thought experiment, it needs to be remembered that Options 1 and 2 are mutually exclusive. Continuing the analysis from the point of view of Region 2, the existence of direct linkages are conditional upon Option 2 being employed while the existence of indirect linkages are conditional upon Option 1. In this context "equality" of effects presumes the comparison of the effectiveness of one set of linkages in the absence of the other, compared to the effectiveness of the other in the absence of the first. Consider the situation where the direct and indirect linkage effects would be equal in this sense. For the following analysis it is useful to represent this as:

$$A_{23} = A_{21}\Delta_1 A_{13} \tag{25a}$$

for the flow of influence from the facility to Region 2 on the one hand and

$$A_{32} = A_{31}\Delta_1 A_{12} \tag{25b}$$

for the flow of influence from Region 2 to the facility on the other, and where the caveat on the meaning of these equalities needs to be kept in mind. (That is, the comparison is really between a situation where LHS (25a) and (25b) exist, and are of a certain size, and a mutually exclusive circumstance where these direct relationships between Regions 2 and 3 do not exist. Instead, in the alternative circumstance the RHS (25a) and (25b) exist but are assumed - in the "equality" thought experiment - to be of the same size as what was previously considered for the LHS of (25a) and (25b) respectively).

In the case where (25a) and (25b) hold as equalities in the sense described above, inspection of (24') and (24") suggests that the difference between the effects on the strength of Region 2's internal multiplier depend entirely upon the conditional indirect multiplier matrix $\Delta_2^{(3)}$.

Now, under assumptions (25a) and (25b), (23') would be equivalent to:

$$\Delta_3^{(2)}\Big|_1 = \left\{I - \left[A_{33} + A_{31}\Delta_1 A_{13} + A_{32}\Delta_2^{(1)}A_{23}\right]\right\}^{-1} \qquad (26)$$

and this is unambiguously greater than (23"). Under these conditions, ceteris paribus, Region 2 should lobby to have the facility located in Region 1.

Now consider the more general case where indirect linkages are "stronger" than direct ones. Before proceeding, four caveats need to be noted. Firstly, meaningful measurement here requires some metric for comparison of the direct and indirect linkages matrices which are displayed respectively on the LHS and RHS of (25a) and (25b). Secondly, these matrices are at least partly hypothetical. Prior to installation of the facility, none of the linkage matrices $A_{13}$, $A_{31}$, $A_{23}$ and $A_{32}$ exist and after installation only two of these four (and only the LHS or the RHS of (25a) and (25b), but not both sides) will exist. However, reasonable projections for all four of the matrices $A_{13}$, $A_{31}$, $A_{23}$ and $A_{32}$ must be provided to enable decisions on infrastructure location to be made. Third, the scenario needs to be faced that the relationship between direct and indirect effects might favour the dominance of one of these over the other in terms of links flowing in the direction of purchasing by the facility from Region 2 – the linkages compared in (25a) – but could at the same time favour the alternative dominance when it comes to considering links flowing from Region 2 to the facility – the linkages compared in (25b). Fourth, within the linkages matrices themselves, it may well be the case that for some sectors in the two regions direct effects dominate while for others indirect effects do so.

These caveats make more pertinent the observation that analytical techniques can only take one so far and that the ultimate implications need to be investigated empirically. Nevertheless, the analytical implications of the model employed in this analysis clearly point to the factors requiring empirical investigation. It is instructive, therefore, to push the analysis one step further. Given these caveats the point may still be made that, starting from assumptions (25a) and (25b), if the most reasonable alternative assumption is that indirect linkages are dominant – that is, that the RHS terms in (25a) and (25b) tend to dominate the LHS terms – then it follows a fortiori from a comparison of $\Delta B[3]_{22}\Big|_1$ in (24') with $\Delta B[3]_{22}\Big|_2$ in (24"), given also that $\Delta_3^{(2)}\Big|_1$ in (23') will clearly dominate $\Delta_3^{(2)}\Big|_2$ in (23"), that Region 2 will benefit more, measured by the effect of the facility on the strength of its internal multiplier, if the infrastructure is located in Region 1.

Of course, the implication that the dominance of indirect effects suggests that the best option is indirect location is not a very startling one. Nevertheless, it is one which suggests that indirect linkages do need to be calculated and compared with direct linkages for their strength. More interesting, however, is the result that under equality of direct and indirect effects the best locational choice will still be the indirect one. In fact, a stronger result is implied. Inspection of (23') shows that there is an additional term, $A_{31}\Delta_1 A_{13}$, which will tend to maintain $\Delta_3^{(2)}\Big|_1$ at greater strength than $\Delta_3^{(2)}\Big|_2$, even for some range in which the direct linkages – exhibited on the LHS of (25a) and (25b) – exceed in strength the alternative indirect linkages which are exhibited on the RHS of these expressions. Thus, there is undoubtedly a range in which the sensible policy option for Region 2 to pursue is to seek location of the new infrastructure in Region 1 even when linkages through direct location would exceed the strength of linkages through indirect location.

## 6. CONCLUSION

This paper has argued that the question of regional location of infrastructure needs to be examined carefully within a multiregional context in which alternative locations are considered in terms of either their direct or indirect linkages to any given region. The ability of an infrastructural development to strengthen a region's internal intersectoral multiplier is not necessarily related to the proximity of the infrastructure but depends crucially on links between the infrastructure and the region in which it is located as well as on regional trading linkages.

A technique of decomposition of the change in the Leontief inverse, which would result from the establishment of some new facility, has been presented and employed to demonstrate the range of factors which determine the outcome.

The type of analysis proposed in this paper has implications for regional development policy. While the analysis conducted here is highly stylised and needs to be interpreted cautiously, nevertheless it seems reasonable to conclude that a regional policy based upon justification of location of a facility because it is "our turn" or "the other region has other facilities" may be counterproductive. This could be especially so if the other region is economically strong and if the region in question is engaging in import substitution policies or promoting "buy local" campaigns which may be inhibiting trade. The conjunction of these conditions is not improbable.

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