REGIONAL INPUT-OUTPUT, LEONTIEF-STROUT AND UNCERTAINTY

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ABSTRACT Balance relations, as inferred from the 'supply and demand pool' assumptions of Leontief and Strout, were used by Wilson in the 70's as technological constraints on a probabilistic multi-regional model of the flow of commodities. The model, based on entropy maximization, was also constrained to reproduce the observed average travel cost per unit of commodity shipped, integrating the technology and gravity effects in the flow determination. In this paper, the Leontief-Strout approach is interpreted in terms of expected data availability. Although such arguments mirror their supply and demand pool assumptions, their own balance relations turn out to be partially inconsistent with these assumptions. The first task of the paper is the development of a consistent set of balance relations. Secondly, these corrected balance relations are used in a new entropy maximization framework, which also allows for regional interdependencies by introducing logistic constraints on regional input capacity. Further extensions of the approach lead to the generation of probabilistic supply functions as tools within a potential CGE analysis. This option requires the introduction of prices, permitting a profit constraint to replace the simple transport cost constraint of the earlier model.

1. INTRODUCTION

The multi-regional input-output approach of Leontief-Strout, described in Leontief and Strout (1963), built on the Chenery-Moses model introduced by Chenery (1953). In this model two key assumptions are embedded (i) the demand pool assumption, whereby the users of intermediate inputs are taken as indifferent to their region of origin and (ii) the supply pool assumption, where producers are taken as indifferent to the region of destination of their outputs. The resulting model turns out to be much more tractable than a full interregional model, and has been widely applied. Leontief and Strout used an independent gravity type model to estimate the resulting trade flows, as a substitute for the provision of trade coefficients in models from the Chenery-Moses tradition. Wilson (1970) provided a more integrated procedure, setting up a conventional entropy framework to determine the commodity flows enhanced by the Leontief-Strout input-output balance relations introduced as constraints. In other words, the flows were co-determined by the transport cost information together with the technological information embodied in the Leontief-Strout representation of input-output. Another enhancement was to replace the entropy framework by one from information theory based on historical trade patterns (Snickars and Weibull, 1977), yielding models such as in Batten (1983).

In Roy (1997), the introduction of uncertainty in the producer profit terms allowed prices to be retained explicitly in a regional input-output framework.

Although the retention of prices is desirable from a theoretical viewpoint, such price information at the regional level is often extremely difficult to obtain. In fact, the whole of regional input-output analysis is plagued by a paucity of survey data. So, in this paper, the main emphasis is to base the multi-regional model of interregional flows on just the survey data obtainable without a major effort. In fact, the two key Leontief-Strout assumptions are entirely equivalent to two conditions based on data availability, as follows (i) the input-output coefficients are just defined at the region of production, independently of the mix of the inputs from other regions and (ii) the flows of commodities between regions are only known in terms of the commodity (viz. sector) being shipped, independently of the sector or sectors to which they are being supplied as intermediate inputs. However, the Leontief-Strout input-output balance relations turn out to be logically incorrect, confusing output with the sum of intermediate input flows. New balance relations are derived, compared with the conventional relations and introduced into an entropy maximization framework, which may determine either interregional or multi-regional flows.

In any short run regional supply model, it is desirable to include input capacity constraints. Of course, an obvious way to achieve this is to introduce $\leq$ inequality constraints on regional capacity for each sector. However, such constraints, when inactive, maintain separability and have no influence whatsoever on the flows - they only factor into the analysis once they become active. Intuitively, this is not very plausible. In most sectors, a 'vintage' distribution over regional capacity exists, and when the capacity limits of a sector are being hard pressed in a certain region, one would expect some spillovers into adjacent regions. In fact the generic logistic forms of regional supply functions motivated by Hotelling (1932) demonstrate this property. In this paper, an additional entropy term recognizing heterogeneity within the available capacity is shown to yield a logistic supply function. This generates a further enhancement to the Wilson framework. Of course, if regional price information were available, our transport cost constraint should be replaced by a short run constraint on profits, allowing our result to truly be interpreted as a supply function. In addition, by use of the special information theory method of Roy (1987), the model not only perfectly reproduces the base period flow distribution (as for conventional information theory models), but is simultaneously responsive to changes in freight prices and the transport network. Technological change can be included by exogenous adjustments to the regional input-output coefficients, with the technological balance relations being imposed anew in the projection time period. Finally, these results are generalized by the introduction of explicit market prices, and compared with the formulation in Roy (1997). Short run profit constraints replace the simple transport cost constraints, and flows are defined in quantity rather than money units. The resulting flow relations then represent logistic supply functions.

2. FORMULATION OF ENHANCED MODELS

2.1 From Interregional to Multi-Regional - Appropriate Balance Relations

In the course of introduction of the new approach, it seems relevant to say something on the respective merits of a 'full' interregional analysis and a multi-regional procedure. Interregional input-output coefficients a_{ij}^{rs} are conventionally defined as

$$a_{ij}^{rs} = x_{ij}^{rs} / X_j^s \tag{1}$$

expressing the quotient of the input x_{ij}^{rs} of sector i from region r into sector j in region s over the total output X_j^s of sector j in region s. Then, the *usage* of the output X_i^r of sector i in region r is expressed as that going to supply intermediate inputs of all sectors into all regions and the amount Y_i^r going to final demand, yielding

$$X_i^r = \sum_{js} x_{ij}^{rs} + Y_i^r \tag{2}$$

We should note that a comparison of the interregional input coefficients a_{ij}^{rs} in region r with a_{ij}^{zs} in another input supply region z yields *no guidance* on the *relative technological effectiveness* of obtaining a unit of output of sector j in region s from a unit of input of sector i from region r *vs.* a unit of the same input from region z. In order to achieve such a comparison, we would need to distinguish the part of the output X_j^{rs} which is produced from inputs of sector i from region r versus those from region z, and have enhanced coefficients $\tilde{a}_{ij}^{rs}$ defined as

$$\tilde{a}_{ij}^{rs} = x_{ij}^{rs} / X_j^{rs} \tag{3}$$

If it is not feasible in practice to obtain such differentiated output information, it seems to this writer that the gains from interregional analysis may be insubstantial. It appears to yield nothing to enable comparison of the technology embedded in the inputs from different regions regarding their output efficiency in different sectors in the same or other destination regions, which is what a true *interregional production function* should be trying to measure between regions[1]. On the other hand, it does allow us to compare the ratios of local inputs vs. the inputs from other regions, specialized for each different producing sector in a region. Thus, although the conventional interregional analysis does not seem capable of identifying the 'transmission' of technology between regions, it can measure the full set of commodity to sector flows between each pair of regions, and try to include the influence of transport costs on these flows. As far as output

[1] Perhaps these points are obvious to the experienced input-output specialist. However, they do not appear to be greatly emphasized in the literature.

technology is concerned, it is most appropriately embedded in the multi-regional coefficients a_{ij}^{s}, denoting the number of units $x_{ij}^{s} = \sum_r x_{ij}^{rs}$ of input of sector i going into sector j in region s required to produce a unit of sector j output in the same region s, given as

$$a_{ij}^{s} = x_{ij}^{s} / X_j^{s} \qquad (4)$$

Of course, units of the input x_{ij}^{s} of sector i used for sector j in region s are supplied from region s itself, as well as potentially from all other regions r.

Referring to Leontief and Strout (1963), it is clear that the simplification of the conventional interregional relations (1) to the multi-regional relations (4) corresponds to their *demand pool* assumption. The total flow $\sum_s x_i^{sr}$ of sector i *into* region r from all regions s (including itself), rather than the output X_i^r, is assumed to be distributed to final demand Y_i^r and as inputs x_{ij}^{r} to all other sectors j (including i itself), leading to

$$\sum_s x_i^{sr} = \sum_j x_{ij}^{r} + Y_i^r \qquad (5)$$

The *supply pool* assumption implies that the outputs of each sector i in each region r are pooled before being allocated to supply regions. Instead of (4) with s replaced by r being substituted for x_{ij}^{r} into (5), output is again replaced by a sum of flows (this time by *out*-flows), yielding the final balance relations

$$\sum_s x_i^{sr} = \sum_j a_{ij}^{r} (\sum_s x_i^{rs}) + Y_i^r \qquad (6)$$

Upon absorbing the regional index s by denoting the sums over s with '•', we obtain (6) in its usual form

$$x_i^{\bullet r} = \sum_j a_{ij}^{r} x_i^{r\bullet} + Y_i^r \qquad (7)$$

However, these relationships are quite unusual, equating a sum of in-flows to equal the flows to supply intermediate inputs plus final demand. Conditions (2), which express *total output* as the sum of flows to supply intermediate inputs plus final demand, are the only relations fully supported by the theory. The challenge is to make them multi-regional rather than interregional, consistent with the availability of technology data just by region of production and flow data between regions just in terms of the commodity (sector) shipped.

If relations (1) are transposed, indices r and s interchanged and the whole summed over j and s, we have

$$\sum_{js} x_{ij}^{sr} = \sum_{js} a_{ij}^{sr} X_j^r \qquad (8)$$

Now making the demand pool assumption, we can set $a_{ij}^{r} = \sum_s a_{ij}^{sr}$ in (7), yielding

$$\sum_{js} x_{ij}^{sr} = \sum_j a_{ij}^r X_j^r \tag{9}$$

Then, the output term can be removed from (9) by substituting for it from (2) in terms of the sum of intermediate plus final demands, giving

$$\sum_{js} x_{ij}^{sr} = \sum_j a_{ij}^r (\sum_{is} x_{ji}^{rs} + Y_j^r) \tag{10}$$

If the commodity flows are just defined as x_i^{rs} in terms of the commodity i being shipped, then the extra indices can be removed from (10), leading to

$$\sum_s x_i^{sr} = \sum_j a_{ij}^r (\sum_s x_j^{rs} + Y_j^r) \tag{11}$$

The final step, as above, is to denote the s summation by a dot, yielding

$$x_i^{\bullet r} = \sum_j a_{ij}^r (x_j^{r\bullet} + Y_j^r) \tag{12}$$

This is clearly seen to be different to the conventional Leontief-Strout condition (7), with the final demand being under the summation and scaled by the multi-regional coefficients. As it has eluded the author to find a flaw in the above reasoning, which proceeds directly from the classical conditions (1) and (2), there seems no other possibility than that the Leontief-Strout result is not correct. As stressed earlier, they abandoned the fundamental relation (2) to somehow express the sum of in-flows as the sum of intermediate demands and final demand, without seeming to realize that the outputs must also contribute to *final demands* in the sectors j to which sector i is an intermediate input. However, as multi-regional analysis seems more usually to be carried out via the trade coefficient approach than via the method of Leontief-Strout (Isard, *et. al.,* 1998), the above error may not be of great concern in practice. However, in our quest for a model recognizing the interdependent influences on flows of both technology and transport costs, it is important to have the technology represented by the correct balance relations.

2.2 An Integrated Multi-Regional Approach

In this section, a considerable enhancement of Wilson (1970) is made, inserting the correct technological balance relations and including logistic constraints on regional input capacity. Also, a strong distinction is made between the estimation form of the model and a transformed version for projection, building on the formalism introduced by Lesse (1982).

Model Estimation

Consider that we have base period data, both on the *capacity* $\widetilde{X}_{ir}^0$ of sector i to supply inputs from region r and the total outflow $\overline{X}^{s0}$ of all sectors into region s. Also, let Y_j^{r0} be the base period final demand for sector j in region r. Finally, let

$\bar{X}^0$ be the total value of all inputs in the system, $\bar{c}^0$ be the average transport cost per value unit of commodity shipped and c_i^{rs0} be the transport cost per value unit of sector i between regions r and s[2]. Then, the number of microstates Z can be given as the number of ways $\widetilde{X}_{ir}^0$ distinguishable capacity units may be divided into $(\Sigma_s x_i^{rs})$ which are utilized and $(\widetilde{X}_{ir}^0 - \Sigma_s x_i^{rs})$ which are unutilized, times the number of ways that the $(\Sigma_s x_i^{rs})$ distinguishable utilized units may be allocated to regions s, yielding

$$Z = \pi_{ir} \{ \widetilde{X}_{ir}^0 ! / [(\widetilde{X}_{ir}^0 - \Sigma_s x_i^{rs})! \, (\Sigma_s x_i^{rs})!]\} \, . \, \pi_{ir} \{(\Sigma_s x_i^{rs})! / (\pi_s x_i^{rs}!)\} \tag{13}$$

After some cancellation, setting the entropy S as the natural log of Z and applying the Stirling approximation, we obtain as our entropy maximization objective

$$S = - \Sigma_{irs} x_i^{rs} (\log x_i^{rs} - 1) - \Sigma_{ir} (\widetilde{X}_{ir}^0 - \Sigma_s x_i^{rs}) [\log(\widetilde{X}_{ir}^0 - x_i^{rs}) - 1] \tag{14}$$

To the above objective, total destination in-flow constraints are imposed as

$$\Sigma_{ir} x_i^{rs} = \bar{X}^{s0} \tag{15}$$

The logistic capacity 'constraint' in the second entropy term in (14) allows the usual origin constraints to be omitted. Now, apply the average transport cost constraint as

$$\Sigma_{irs} x_i^{rs} c_i^{rs0} = \bar{c}^0 \, \bar{X}^0 \tag{16}$$

Finally, for the flow pattern to be consistent with our multi-regional technology, our new input-output balance relations (11), not in conflict with (15), are applied as

$$\Sigma_s x_i^{sr} - \Sigma_j a_{ij}^{r0} \Sigma_s x_j^{rs} = \Sigma_j a_{ij}^{r0} Y_j^{r0} \tag{17}$$

With final demand taken as exogenous input, as for the conventional analysis, the right-hand side of (17) is a known value, as required. Applying Lagrangian theory, (14) is maximized under (15) with multipliers λ_s, (16) with multiplier β and (17) with multipliers α_{ir}. Differentiation with respect to x_i^{rs} and equating to zero gives

$$x_i^{rs} = (\widetilde{X}_{ir}^0 - \Sigma_s x_i^{rs}) \exp - [\lambda_s + \beta c_i^{rs0} + \alpha_{is} - (\Sigma_j \alpha_{jr} a_{ji}^{r0})] \tag{18}$$

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[2] By normalizing with respect to prices, transport costs could be converted into *quantity* units

Upon transposing, the explicit logistic result emerges

$$x_i^{rs} = \widetilde{X}_{ir}^{0} \; exp - [\lambda_s + \beta c_i^{rs0} + \alpha_{is} - (\textstyle\sum_j \alpha_{jr} a_{ji}^{r0})] \,/\, \{ 1 + (\textstyle\sum_s exp - [\lambda_s + \beta c_i^{rs0} + \alpha_{is} - (\sum_j \alpha_{jr} a_{ji}^{r0})]\} \tag{19}$$

Setting $A_s = exp - \lambda_s$, $B_{ir} = \widetilde{X}_{ir}^{0} / \{1 + (\sum_s A_s D_{is} E_{ir} exp - \beta c_i^{rs0})\}$, $E_{ir} = \pi_r (D_{jr})^{a_{ji}^{r0}}$ and $D_{is} = exp - \alpha_{is}$, we can substitute into the constraints. With B_{ir} and E_{ir} already given explicitly above in terms of the other unknowns, we merely need to give expressions for A_s and D_{ir}, which after setting $F_{is} = (\sum_r B_{ir} E_{ir} exp - \beta c_i^{rs0})$, come out as

$$A_s = \overline{X}^{s0} / \textstyle\sum_i (D_{is} F_{is}) \; ; \; D_{ir} = \{ \sum_j a_{ij}^{r0} [Y_j^{r0} + B_{jr} E_{jr} \sum_s A_s D_{js} exp - \beta c_j^{rs0}]\} / F_{ir}] \tag{20}$$

It is noticed that the expression for D_{ir} is a function of itself, requiring an iteration within an iteration. The gravity parameter β can be determined simultaneously using linear extrapolation. Alternatively, the non-linear equations for both D_{ir} and β could be solved simultaneously using the Newton-Raphson approach, with those for A_s, B_{ir} and E_{ir} being solved using the conventional successive substitution technique. Convergence is guaranteed (if data is consistent) for this problem with a strictly concave objective and linear constraints. The flows are then given simply as

$$x_i^{rs} = A_s B_{ir} D_{is} E_{ir} exp - \beta c_i^{rs0} \tag{21}$$

Although (21) may appear to be a *separable* function, connectivity is embedded in the relations (20). In fact, the above solution process illustrates the *interdependencies* between transport costs, production technology and logistic capacity constraints in the determination of the flows. After the commodity flows x_i^{rs} are obtained from the above, the regional *outputs* X_i^r can be found from the multi-regional form of (2) : $X_i^r = \sum_s x_i^{rs} + Y_i^{r0}$. Also, relation (4) allows the *intraregional, intersectoral* flows x_{ij}^s to be evaluated, consistent with the multi-regional technology. Thus, our model yields intersectoral flows within regions and flows of a given commodity (sector) between regions. At this estimation step, the modelled flows x_i^{rs} should be compared with the observed base period flows x_i^{rs0} using a goodness of fit measure, such as Root Mean Square. If the fit is good, the entropy approach will be adequate, especially if the goodness of fit of the model is also accompanied by a significant reduction in the uncertainty of the distribution. Otherwise, one should adopt the information theory procedure described later.

Models in the tradition of Chenery (1953) were able to apply trade coefficients to yield *interregional flows*, despite the limitations of a multi-regional technology. The above approach can be modified to do the same. If a more disaggregated form of the balance constraints (10) is applied, with the summation over j removed, giving

$$\sum_s x_{ij}^{sr} = a_{ij}^{r0} \left(\sum_{is} x_{ji}^{rs} + Y_j^{r0}\right) \tag{22}$$

it is easy to show that the multi-regional model (19) has the interregional analogue

$$x_{ij}^{rs} = \widetilde{X}_{ir}^{0} \exp - [\lambda_s + \beta c_i^{rs0} + \alpha_{ijs} - \alpha_{jir} a_{ji}^{r0}] / \{ 1 + \left(\sum_{js} \exp -[\lambda_s + \beta c_i^{rs0} + \alpha_{ijs} - \alpha_{jir} a_{ji}^{r0}]\right\} \tag{23}$$

where the Lagrange multipliers α_{ijs} have one more dimension than those in (19). Although no further technological information is provided over (19), we can now obtain information on the regional mix of inputs.

Use of Model for Projection

The main task in transforming the estimation model formulation into a form suitable for projection is to choose the Lagrange multipliers which should be treated as parameters in projection, and those which must be evaluated anew. This clearly relates to which information should reasonably be treated as exogenous input and which treated as endogenous output. In terms of the structure of the model, it is considered that the exogenous input should include any quantity changes, such as changed regional input capacities $\widetilde{X}_i^{r0}$ and changed final demand Y_i^r. If transport costs change to c_i^{rs}, the availability as a parameter of the gravity Lagrange multiplier β allows the influence of these changes to be assessed by the model (Lesse, 1982). In addition, any new multi-regional I-O coefficients a_{ij}^r must be provided exogenously. Finally, the treatment of the Lagrange multiplier λ_s as a parameter allows the outflows to be endogenous in projection, in response to the changes in input capacities, final demand, transport costs and the multi-regional technology. In this case, the projection form of (19) changes to

$$x_i^{rs} = \widetilde{X}_i^{r} \exp - [\lambda_s + \beta c_i^{rs} + \alpha_{is}' - (\sum_j \alpha_{jr}' a_{ji}^r)] / \{ 1 + \left(\sum_s \exp - [\lambda_s + \beta c_i^{rs} + \alpha_{is}' - (\sum_j \alpha_{jr}' a_{ji}^r)]\right\} \tag{24}$$

with merely the multipliers α_{is}' having to be re-evaluated to satisfy an updated form of the balance relations (17). The evaluation of these multipliers would merely entail successive substitution in the relations associated with (20), but where A_s and β are now given exogenously. This recursive substitution process retains the non-separable structure of the model. Furthermore, it is clear that our probabilistic approach has bypassed the necessity of matrix inversion, which is routinely required for the deterministic procedure.

An Adjustment From Information Theory

As stressed in Batten (1983), the more general information theory approach may give improved prediction ability for this class of problem. If the goodness of fit of the estimated model is not satisfactory, the projection model is likely to yield improved results if information bias terms are computed and inserted into the model, as demonstrated via an information theory procedure in Roy (1987). We proceed as follows. If the estimated entropy model (19) yields $\bar{x}_i^{rs}$ as the modelled flows, we compute ratios z_i^{rs} as the quotient $(x_i^{rs0} / \bar{x}_i^{rs})$ of the observed and modelled flows and z_i^r as the quotient $(\widetilde{X}_i^{r0} - \sum_s x_i^{rs0}) / (\widetilde{X}_i^{r0} - \sum_s \bar{x}_i^{rs})$ of the observed and modelled unutilized capacities. The next step is to normalize the *z's* as probabilities $q_i^{rs} = z_i^{rs} / (\sum_{irs} z_i^{rs})$ and $q_i^r = z_i^r / (\sum_{ir} z_i^r)$. If the *q's* are applied as prior probabilities in the denominator of the log terms in the entropy objective (14) and this revised estimation problem is solved anew, it is demonstrated in Roy (1987) that (i) all the Lagrange multipliers are *invariant* to this changed objective and (ii) the modelled flows *precisely* correspond to the observed flows x_i^{rs0}. Of course, this is a good basis for projection. In addition, the outflow and gravity parameters are retained, with the latter allowing the model to be sensitive to future transport cost and network changes. The consistency of the flows with the multi-regional technology available at the projection period is assured by applying the technological balance relations anew. The projection relations (24) now become

$$x_i^{rs} = \widetilde{X}_i^r \; q_i^{rs} \exp - [\lambda_s + \beta c_i^{rs} + \alpha_{is}'' - (\textstyle\sum_j \alpha_{jr}'' a_{ji}^r)] / \{ q_i^r + (\textstyle\sum_s q_i^{rs} \exp - [\lambda_s + \beta c_i^{rs} + \alpha_{is}'' - (\sum_j \alpha_{jr}'' a_{ji}^r)]\} \quad (25)$$

where the Lagrange multipliers α_{is}'' associated with the new input-output balance constraints will change from those in the pure entropy formulation (24). It is suggested that (25) be used in place of (24) for most applications.

Some Comparisons

The approach presented above integrates technology changes and transport network and cost changes in the evaluation of multi-regional flows. As a probabilistic model, it fits parameters to observations, rather than relying on deterministic optimization. The logistic form of the supply relationships promotes spillovers into adjacent regions when there is a high pressure on capacity in a given region. At the same time, the model doesn't attempt to project changes in technology. It is the user's responsibility to provide any changed I-O coefficients a_{ij}^r. Its primary aim is to estimate the influence on regional flows and output when exogenous changes in input capacities, final demand, transport costs and technical coefficients are introduced. It will need to be tested with conventional multi-regional data to verify its suitability for these tasks. This data will need to be enhanced by capacity data for each sector in each region. Without the technological balance relations, the model would resemble a commodity flow model enhanced by the logistic input capacity constraints. However, we maintain that the flows must be consistent with the multi-regional technology being used, justifying the enforcing of these balance relations. The Leontief-Strout balance relations, which were also used by Wilson (1970), are shown to be not fully grounded in the basic relations expressing the usage of output to be the sum of supplies to intermediate inputs and final demand.

What If We Have No Sector Input Capacity Data ?

Data on the sector input capacities $\widetilde{X}_i^{r0}$ for each sector in each region may be quite difficult to acquire. In that case, base period constraints are introduced on the total value $\overline{X}^{r0}$ of out-flows from each origin region r, in the form

$$\sum_{is} x_i^{rs} = \overline{X}^{r0} \tag{26}$$

If the capacity entropy is omitted as the second term in (14), and (26) attached with a multiplier η_r, then the previous solution (19) is replaced by

$$x_i^{rs} = exp - [\lambda_s + \eta_r + \beta c_i^{rs0} + \alpha_{is} - (\sum_j \alpha_{jr} a_{ji}^{r0})] \tag{27}$$

With A_s, D_{is} and E_{ir} defined as below (19) and B_r defined as $exp - \eta_r$, we can set up the following set of recursive relations

$$A_s = \overline{X}^{s0} / \sum_{ir} B_r D_{is} E_{ir} exp - \beta c_i^{rs0} \qquad B_r = \overline{X}^{r0} / \sum_{is} A_s D_{is} E_{ir} exp - \beta c_i^{rs0}$$
$$D_{ir} = \{\sum_j a_{ij}^{r0} [Y_j^{r0} + B_r E_{jr} \sum_s A_s D_{js} exp - \beta c_j^{rs0}]\} / \{\sum_s A_r B_s E_{is} exp - \beta c_i^{sr} \tag{28}$$

These three relations, together with that for E_{ir}, can be solved iteratively for the unknown Lagrange multipliers, with β being obtained, as before, via linear extrapolation. The final relations are identical to (21) except for B_r replacing B_{ir}. For projection, A_s, B_r and β are treated as parameters, with the recursive relations for D_{is} and E_{ir} providing the interdependencies associated with the inverse in the conventional solution. Clearly, the information theory adjustment should again be used when the fit to the base period flows is not adequate.

Some Further Developments

In Roy (1997), a probabilistic approach was developed which retained prices in input-output analysis. Although regional prices are not easily obtained, the provision of prices opens the opportunity for input-output models to be developed as short run supply functions, allowing tâtonnement with a compatible demand model in a regional CGE framework. The main difference with the foregoing is that the ***supply to final demand*** is now obtainable as an ***output*** from our supply model, rather than being inserted as an ***input*** via our balance relations, which must be omitted here. The first change to the above model is that the flows x_{ij}^{rs} are defined as *interregional* in terms of *quantities* shipped, rather than in terms of dollars. Whereas the objective function (14) remains unchanged in form, the outflow constraints (15) are summed over i and r. The transport cost constraint (16) should now be modified to a constraint on short run profits, reflecting the classical objective of the competitive firm. If $\bar{\pi}^0$ is taken as the base period unit profit per dollar of all commodities produced, then we should replace (16) by a profit constraint in terms of revenue minus costs, yielding

$$\sum_{ijrs} x_{ij}^{rs} p_i^{r0} / (m\, a_{ij}^{s0}) - \sum_{ijrs} (p_i^{r0} + c_i^{rs0})\, x_{ij}^{rs} = \bar{\pi}^0 V^0 \tag{29}$$

where V^0 are the total dollars of production, m the number of row sectors and the *p's* the fob prices in the base period. Also, assuming that the input-output coefficients a_{ij}^{s0} are given (as usual) in value terms, the output prices p_j^{s0} cancel out in the above revenue expression. In this term, the output has been expressed as the *average* over all rows of the quotients of input divided by the regional coefficient, which emerge as constant values in the deterministic theory (Roy, 1997). If the Lagrange multiplier on the profit constraint (29) is taken as γ, then constrained maximization of (14) yields

$$x_{ij}^{rs} = \tilde{X}_i^{r0} \exp - [\lambda_{is} - \gamma\{(p_i^{r0}/m\, a_{ij}^{s0}) - p_i^{r0} - c_i^{rs0}\}] / \{1 + \sum_{js} \exp - [\lambda_{is} - \gamma\{(p_i^{r0}/m\, a_{ij}^{s0}) - p_i^{r0} - c_i^{rs0}\}]\} \tag{30}$$

This truly represents a logistic probabilistic supply function, where the profits are included as a constraint based on observations, rather than as the objective in the deterministic theory. Output can be obtained as an average of the quotients of intermediate inputs and regional coefficients, consistent with their role in the revenue relations. The supply to final demand can then be computed from (2). Thus, if regional market prices are available, the relations (30) should be used in preference to (23), where final demand is exogenous and prices have been 'absorbed', not being capable of modification between the base period and the projection period.

The overall aim of the paper has been to extend the conventional deterministic theory to be probabilistic. In addition, we can identify the interdependencies between logistic input capacity constraints, production technology and transport costs, in their influence on the pattern of multi-regional or interregional flows. When regional prices are included, regional supply functions are produced, allowing the supply to final demand to be endogenous. At the same time, an apparent error in the input-output balance relations of the classical multi-regional model has been uncovered. Much empirical work remains to be done to demonstrate the relevance of the proposed alternative model structures.

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